Christmas Catastrophe!

Teacher Notes

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Strand: Number and Algebra
Group: Number Theory, Sequences
Suggested Age: 10 - 16

If using this worksheet with lower ability pupils provide them with the added hint of:

"Try doing the sum with a lower number than the maximum of 100 bulbs. Repeat this with a couple of numbers and see if you can spot a pattern"

There are many different ways of finding the solution. Pupils should be encouraged to try different methods in order to find the quickest. If the pupils have found the answer one way, ask them if this gives a clue to an alternative quicker way.

Solution

The simplest solution is to realise that this sum is equivalent to 100^2, so the answer is **10,000**.

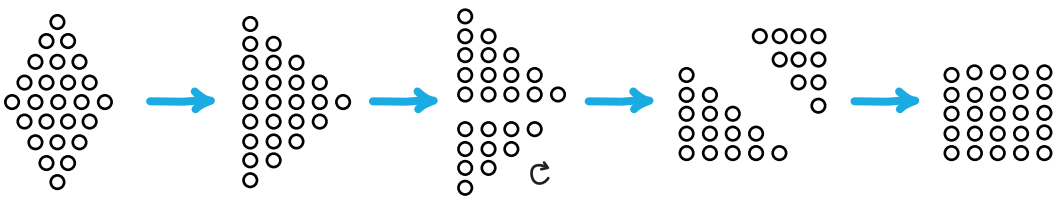
This can be shown by looking at smaller numbers first. Say the biggest row of bulbs had the following number of bulbs in it, then the sums would look like:

3 bulbs max = 1 + 2 + 3 + 2 + 1 = 9 bulbs total
4 bulbs max = 1 + 2 + 3 + 4 + 3 + 2 + 1 = 16 bulbs total
5 bulbs max = 1 + 2 + 3 + 4 + 5 + 4 + 3 + 2 + 1 = 25 bulbs total

To make this clearer still you can rearrange the sums:

$$1 + 2 + 3 + 4 + 5 + 4 + 3 + 2 + 1 = (1 + 4) + (2 + 3) + (3 + 2) + (4 + 1) + 5$$
$$= 5 + 5 + 5 + 5 + 5$$

Pictorial Solution:



Other methods of finding the solution are shown on the next page.

Alternative (slower) methods

Summing Arithmetical sequence:

The problem can be simplified to summing 2 sequences from 1 to 99 and adding on the 100 at the end.

So... $1+2+3+\ldots+99+100+99+\ldots+3+2+1 = 2(\overbrace{1+2+3+\ldots..+99}^{\text{Sum (1 to 99)}}) + 100$

Sum of Arithmetical Sequence $= \frac{n}{2}(2a + (n-1)d)$

so Sum (1 to 99) $= \frac{99}{2}(2 + (98)1) = 49.5(100) = 4950$

Total Sum $= (2 \times 4950) + 100 = 9900 + 100 = 10{,}000$

Triangular Numbers:

This problem can be thought of like the 100th triangular number + 99th triangular number.

So the formula for a triangular number can be used: $T_n = \frac{n(n+1)}{2}$

So... $T_{100} + T_{99} = \frac{100(100+1)}{2} + \frac{99(99+1)}{2} = 5050 + 4950 = 10{,}000$