



Year 10 & 11 Scheme of Work

This example shows our exemplification of Year 10 and 11.
Complete Mathematics contains full exemplification for
all year groups from Year 1 upwards.



Mathematics. Solved.

KEY

Unit

STRAND



Curriculum Statement

STRAND



Objective

Objective Description

Topic

NON-STATUTORY

FOUNDATION

HIGHER

Topic Structure

STRAND

Topic

Topic

Topic

NUMBER

Fractions

Fractions (Including Decimals)

Fractions (Including Decimals and Percentages)

Number and Place Value

Powers and Roots

Properties of Numbers

Ratio and Proportion

Ratio, Proportion and Rates of Change

ALGEBRA

Expressing Relationships

Using Equations and Functions

Using Equations, Functions and Graphs

GEOMETRY AND MEASURES

Geometry

Measures

Position and Direction

Properties of Shapes

PROBABILITY AND STATISTICS

Probability

Interpreting Data

Identify and work with fractions in ratio problems

NUMBER

2

Ratios as Fractions

Ratio and Proportion

The objective is for students to understand the meaning of ratio notation and interpret ratio as a fraction. Students should be able to solve simple problems representing ratios as fractions in context. For example: • The ratio of right-handed to left-handed students is 18:2. What fraction of students is left-handed? • The ratio of red counters to blue counters is 4:5. The number of blue counters is doubled. What fraction of the counters are now blue? • If a line is divided in the ratio 1:3, what fraction of the whole line does the smaller part represent? • A bag contains green and yellow counters in the ratio 4:5. What is the probability of randomly choosing a yellow counter?

FOUNDATION

HIGHER

Solving Problems using Ratios as Fractions (including Geometrical)

Ratio and Proportion

The objective is for students to solve problems involving ratios as fractions. Students should be able to represent ratios as fractions, including in the context of geometrical problems. For example: • A bag contains 100 sweets that are either hard-boiled, chocolate, or chewy sweets. $\frac{3}{5}$ are hard-boiled sweets. The ratio of chewy sweets to chocolates is 3:1. How many sweets are chocolate? • Given similar shapes, students can identify the scale factor of enlargement. • Given a scale drawing of a map, students can calculate distances between two cities.

FOUNDATION

HIGHER

Compare lengths, areas and volumes using ratio notation and/or scale factors; make links to similarity

GEOMETRY AND MEASURES

2

Comparing Lengths using Ratios and Scale Factors

Measures

The objective is for students to compare lengths using ratios and scale factors. Students should be able to draw and identify similar shapes using scale factors and ratio notation. For example: • Given a 4cm x 3cm rectangle and a scale factor of 3.5, students can draw an enlarged rectangle and identify the ratio of the perimeters as 2:7. • Given 2 similar shapes and a scale factor, students can calculate missing dimensions. • Given the height of a student, estimate the height of a tree.

FOUNDATION

HIGHER

Comparing Area and Volume of Similar Shapes

Measures

The objective is for students to compare area and volume of similar shapes. Students should be able to identify changes in area and volume of similar shapes using scale factors and ratio notation. For example: • A triangle with an area of 15cm^2 is enlarged by a scale factor of 3. What is the area of the new triangle? • Given the areas of 2 similar triangles, what is the ratio of the area of the smaller triangle to the area of the larger triangle? • Given a solid with a volume of 20cm^3 and a similar solid of volume 160cm^3 , what is the scale factor of enlargement? • Given 2 similar solids, the dimensions of the larger and a ratio of the volumes, students can find the dimensions of the smaller solid.

FOUNDATION

HIGHER

Write equivalent ratios including unitary ratios and the use of the unitary method

NUMBER

2

Equivalent & Simplifying Ratios

Ratio and Proportion

The objective is for students to solve problems involving ratios. Students should be able to use equivalent ratios, simplifying ratios and expressing ratios in the form 1:n as tools to solve problems. For example: • A recipe calls for 30g of butter to make 10 scones. How much butter is needed to make 30 scones? • Mrs Smith's class has 18 boys and 12 girls. Express the ratio of boys to girls in simplest form. • Express the ratio 6:4 in the form 1:n • On a map a length of 8 cm represents a real-life measurement of 24 km. What is the scale of the map in the form 1:n?

FOUNDATION

HIGHER

Solve Problems Using the Unitary Method

Ratio and Proportion

The objective is for students to understand and use the unitary method to solve problems involving ratio and proportion. Students should be able to identify when the unitary method should be used and apply it. For example: • A recipe calls for 120 g of butter to make 12 biscuits. How much butter is needed to make 30 biscuits? • Divide £80 in the ratio 2:3:5 • Farhana and Susie divide £300 in the ratio 2:4. Susie gives $\frac{1}{3}$ of her share to Keisha. Farhana gives $\frac{1}{2}$ of her share to Keisha. How much money did Keisha receive?

FOUNDATION

HIGHER

Understand that X is inversely proportional to Y is equivalent to X is proportional to $\frac{1}{Y}$; {construct and} interpret equations that describe direct and inverse proportion

NUMBER

2

Direct Proportion

Ratio, Proportion and Rates of Change

The objective is for students to understand that proportion represents a multiplicative relationship. Students should be able to understand that direct proportion can be represented as an equation of the form $y=kx$ where k is the constant of proportionality (and k does not equal zero). Higher students should be able to construct and use the equation $y=kx$ to solve problems involving direct proportion. They should be able to express a relationship as a proportion and deduce the constant of proportionality in order to write a formula that can be used to solve problems involving direct proportion. For example: • Given that x is directly proportional to y , what happens to y when x is doubled? • (Higher) 25 metres of ribbon costs £32, how much does 40 metres of the same ribbon cost?

FOUNDATION

HIGHER

Inverse Proportion

Ratio, Proportion and Rates of Change

The objective is for students to understand that inverse proportion can be represented as an equation of the form $y=\frac{k}{x}$ where k is the constant of proportionality and to use this fact to solve problems involving inverse proportion. Students should recognize the equivalence of $y=\frac{k}{x}$ and $x=\frac{k}{y}$. For example, given that x is inversely proportional to y , what happens to y when x is doubled and what happens to x when y is doubled? Higher students should be able to express a relationship as a proportion and deduce the constant of proportionality in order to construct a formula that can be used to solve problems involving inverse proportion. For example: • The number of hours, h , taken to build a wall is inversely proportional to n , the number of workers. If it takes 15 hours for 3 workers to build the wall, how long will it take 8 workers? • Time=Distance/Speed. What happens to time if speed is doubled?

FOUNDATION

HIGHER

Interpret the gradient of a straight line graph as a rate of change; recognise and interpret graphs that illustrate direct and inverse proportion

NUMBER

1

Ratios and Graphs

Ratio, Proportion and Rates of Change

The objective is for students to understand and use the gradient of a straight-line graph in relation to ratio. Students should be able to recognise the gradient of a straight-line graph as a rate of change. They should also be able to recognise, draw and interpret graphs of both direct and inverse proportion. For example, given a conversion graph, students can find the conversion factor using the gradient.

FOUNDATION

HIGHER

Calculate percentage change including reverse percentage problems

NUMBER

2

Percentage Change

Fractions (Including Decimals and Percentages)

The objective is for students to calculate percentage change. Students should be able to find both percentage increase and decrease. For example, students can calculate the new price of a sale item given its original price and the percentage discount.

NON-STATUTORY

Reverse Percentage Change

Fractions (Including Decimals and Percentages)

The objective is to build on students' understanding of percentage change in order to calculate reverse percentage change. Students should be able to find the original value of a quantity given the percentage change. For example, given a sale price and the percentage discount, students can calculate the original price.

FOUNDATION

HIGHER

Work with repeated percentage change including compound interest

NUMBER

2

Repeated Percentage Change

The objective is for students to be able to solve problems involving repeated percentage change. Students should be able to calculate repeated percentage increase and decrease. For example:

- Students can calculate the increasing value of a house over time.
- Students can calculate the depreciating value of a car over time.

NON-STATUTORY

Fractions (Including Decimals and Percentages)

Compound Interest

The objective is to build on students' understanding of repeated percentage change to be able to calculate compound interest. Students should be able to use multipliers and the formula for compound interest. For example, given an opening account balance and interest rate, students can calculate the amount in the account after a given time period.

FOUNDATION

HIGHER

Fractions (Including Decimals and Percentages)

Understand and use scale factors with length, area and volume

GEOMETRY AND MEASURES

1

Scale Factors

The objective is for students to understand and use scale factors with length, area and volume. Students should be able to make the link between enlarged lengths, area and volume using the scale factor. For example, given a cuboid and a scale factor, students can calculate the new dimensions, surface area and volume of the enlarged cuboid.

NON-STATUTORY

Measures

Deduce expressions to calculate the nth term of linear sequences.

ALGEBRA

2

Generating sequences

Expressing Relationships

The objective is to consolidate and build upon students' skills in generating linear sequences. Students should be able to generate sequences from a term-to-term rule, nth term rule and given diagrams. They should also be able to find the value of any value number in the sequence when given its position in the sequence. For example: • A sequence starts with 2 and has the term-to-term rule of double and add 3. Write the first 5 terms of the sequence. • Given the first three patterns of a sequence, students can draw the next two patterns. • Find the 100th term in the sequence $3n - 4$. • A sequence has nth term rule $4n - 3$. What term has the value of 77?

FOUNDATION

HIGHER

Finding the nth term

Expressing Relationships

The objective is for students to find nth term rules of a linear sequence. Students should be able to find the nth term rule for a given linear sequence in number or diagrammatical form. They should also be able to identify whether a given number belongs to a sequence of numbers, when only given the first few terms or when given the nth term; these can be described using basic number facts or shown algebraically. For example: • Find the nth term of the sequence 5, 8, 11, 14, ... • A sequence begins 11, 15, 19, 23, ... Is 42 in the sequence? Explain.

FOUNDATION

HIGHER

Recognise and use sequences of triangular, square and cube numbers, simple arithmetic progressions, Fibonacci type sequences

ALGEBRA

2

Sequences involving Triangular, Square and Cube Numbers

Expressing Relationships

The objective is for students to recognise and use sequences of triangular, square and cube numbers. Students should be able to continue a given sequence and find a term given the nth term rule. For example: • Write down the next two terms of the following sequence: 3, 12, 27, 48 • Draw the first five diagrams of the triangular numbers and find the next two triangular numbers. • Find the 8th term in the sequence with nth term rule $5n^3$.

FOUNDATION

HIGHER

Other Sequences

Expressing Relationships

The objective is to introduce students to other sequences. Students should be able to solve problems involving simple arithmetic progressions and Fibonacci-type sequences (with the rule given). They will be expected to continue sequences of this nature, when given the first few terms or the rule. For example: • The terms of the Fibonacci sequence are found by summing the two previous terms. List the first 5 terms. • An arithmetic progression starts with $2+b$; $2+3b$; $2+5b$; $2+7b$. The third term is 17. Work out the values of b and work out the nth term rule.

FOUNDATION

HIGHER

Unit 10.3 - Probability calculations for more than one event

PROBABILITY AND STATISTICS

12
HOURS

Apply the property that the probabilities of an exhaustive set of mutually exclusive events sum to one

PROBABILITY AND STATISTICS

1

Mutually Exclusive Events

Probability

The objective is for students to understand and use properties of mutually exclusive events. Students should be able to identify and give examples of mutually exclusive events. Students should also be able to calculate probabilities of mutually exclusive events by knowing and understanding that the sum of the probabilities of an exhaustive set of mutually exclusive events is 1. For example:

- A regular six-sided die is rolled. Are rolling a 2 and rolling a 4 mutually exclusive events? Is rolling a 2 and an even number mutually exclusive?
- A spinner has four sections that are red, green, yellow, and blue. $P(\text{red})=0.3$, $P(\text{green})=0.2$, $P(\text{yellow})=0.16$. Find the probability of blue.
- The probability that Martin completes his maths homework is 0.85. What is the probability that he does not complete his maths homework?

FOUNDATION

HIGHER

Use a probability model to predict the outcomes of future experiments; understand that empirical unbiased samples tend towards theoretical probability distributions, with increasing sample size

PROBABILITY AND STATISTICS

2

Making Estimations with Probability

Probability

The objective is for students to be able to estimate the likelihood of events using probability. Students should be able to calculate estimations of probability based on the frequency of outcomes and events. They should also understand that reliability of results increases with the size of the sample and that there is no element of "memory" in random events. For example, Frankie flips a coin 10 times. It lands on tails 8 times. Can she conclude that the coin is biased? How could she improve her experiment?

FOUNDATION

HIGHER

Relative Frequency Diagrams

Probability

The objective is for students to be able to use and draw relative frequency diagrams. Students should be able to use these diagrams to solve problems involving estimations with probabilities. Students should understand that a relative frequency diagram may show a settling trend as sample size increases and understand how this is useful in estimating a probability. For example, given the results of an experimental trial, students can draw a relative frequency diagram and estimate the probability of an event from the diagram.

FOUNDATION

HIGHER

Unit 10.3 - Probability calculations for more than one event

PROBABILITY AND STATISTICS

12
HOURS

Calculate the probability of independent and dependent combined events, including using tree diagrams and other representations, and know the underlying assumptions

PROBABILITY AND STATISTICS

5

The Addition Rule

Probability

The objective is for students to understand and use the addition rule of probabilities. Students should be able to both use the addition rule and determine when its use is appropriate. For example, students can calculate the probability of rolling a five or a six on a fair six-sided die.

FOUNDATION

HIGHER

The Multiplication Rule

Probability

The objective is for students to understand and use the multiplication rule of probabilities. Students should both be able to use the multiplication rule and determine when its use is appropriate. For example, students can calculate the probability of rolling a six twice when a single die is thrown 2 times.

FOUNDATION

HIGHER

Dependent & Independent Events

Probability

The objective is for students to understand and be secure with the difference between dependent and independent events. Students should be able to calculate the probabilities of dependent events and should know what it means when an event is described as independent. They should understand the implications of problems with or without replacement. For example, students can calculate the probabilities as items are pulled from a bag both with and without replacement.

FOUNDATION

HIGHER

Probability Tree Diagrams

Probability

The objective is for students to draw tree diagrams. Students should be able to draw tree diagrams to show the possible outcomes of independent or dependent events. For example, students can draw a tree diagram to show a train arriving late or on time at various points on its journey.

FOUNDATION

HIGHER

Solving Probability Problems

Probability

The objective is to build on students' ability of drawing tree diagrams to now to use them to calculate the probabilities. Students should be able to draw and use tree diagrams of independent or dependent events. For example, students can draw and use a tree diagram to show the probability of the train arriving on time or late to its first three stops and calculate the probability of the train arriving late to at least one stop.

FOUNDATION

HIGHER

Unit 10.3 - Probability calculations for more than one event

PROBABILITY AND STATISTICS

12
HOURS

{Calculate and interpret conditional probabilities through representation using expected frequencies with two-way tables, tree diagrams and Venn diagrams}.

PROBABILITY AND STATISTICS

2

Venn Diagrams and Probability

Probability

The objective is for students to use Venn diagrams as a method for calculating conditional probabilities. Students should be able to construct a Venn diagram and then answer questions related to probabilities that may be calculated using the diagram. For example: In a survey of preferences between strawberry and chocolate ice cream, 80% of survey responders like chocolate ice cream and 35% like chocolate and strawberry ice cream. What is the probability that a person chosen at random who likes chocolate also likes strawberry?

HIGHER

Probability in Two-Way Tables

Probability

The objective is for students to use two-way tables as a method for calculating conditional probabilities. Students should be able to interpret and construct two-way tables. For example, given a two-way table of models and colours of cars, students can calculate the probability of selecting two cars of the same colour and calculate the probability of one colour given the model of the car.

HIGHER

Apply systematic listing strategies, {including use of the product rule for counting}

PROBABILITY AND STATISTICS

2

Listing Outcomes of Events

Probability

The objective is for students to be able to systematically list the outcomes of exhaustive, independent events and to calculate probabilities. Students should be able to list all permutations and combinations of outcomes. Higher students should be able to know and understand why if there are x ways to do task 1 and y ways to do task 2, then there are xy ways to do both tasks in sequence. For example, • A bag contains chocolates, hard-boiled sweets and chewy sweets. John selects two sweets at random. List the possible pairs and find the probability that the 2 sweets are chocolate. • (Higher) A computer store sells 10 different computers, 4 different keyboards and 2 different mice. How many different combinations of a computer, a keyboard and a mouse are there? Janelle chooses a computer. How many combinations are there now?

FOUNDATION

HIGHER

Probability Sample Spaces

Probability

The objective is for students to construct and use probability sample spaces. Students should be able to complete sample spaces as a method for recording the possible outcomes of exhaustive, independent events and to calculate probabilities. They should be able to display all permutations and combinations. For example, students can complete a sample space to show the outcomes of throwing two fair dice and calculate the probability of rolling the same number on both dice.

FOUNDATION

HIGHER

Work to appropriate accuracy including rounding to decimal places and significant figures

NUMBER

1

Identify and Round to
Given Degrees of Accuracy

The objective is to work with appropriate degrees of accuracy. Students should be able to identify to how many significant figures a number has been given. Students should also be able to round a number to a given number of significant figures or decimal places. For example students should identify that 3500 has been written to 2 significant figures and be able to round 3410 to 1 significant figure.

NON-STATUTORY

Number and Place Value

Apply and interpret limits of accuracy when rounding or truncating, {including upper and lower bounds}.

NUMBER

2

Identify and Round to
Appropriate Degrees of
Accuracy

The objective is to select and round to an appropriate degree of accuracy. Students should be able to interpret scales on a range of measuring instruments and identify or suggest an appropriate degree of accuracy for a measurement from the context of the situation. For example, if working with large masses, students should be able to understand that recording results to the nearest gram is unnecessary

FOUNDATION

HIGHER

Number and Place Value

Accuracy of
Measurements

The objective is to understand that measurements given to the nearest whole unit may be inaccurate by up to one half in either direction. Students should be able to find the largest and smallest possible values for the measurements. Higher students should be able to perform calculations taking into account the limits of accuracy of rounded numbers. For example, • The length of a table is measured to be 1.2 metres. What is its largest and smallest possible length? • (Higher) Ilham travels 2.1 km to school in 15 minutes. What is her minimum and maximum speed?

FOUNDATION

HIGHER

Number and Place Value

Calculate with roots, and with integer {and fractional} indices

NUMBER

3

Calculations Involving Roots

The objective is for students to perform calculations with square and cube roots. Students should be able to recall the first fifteen square numbers, the first five cube numbers and their associated roots. Students should be able to calculate and recognise powers of 2, 3, 4, 5 and 10. Students should also understand that a square root has both positive and negative roots. For example, students should be able to calculate: $\sqrt{49}$ $\sqrt{5} \times \sqrt{5}$ $\sqrt{5} \times \sqrt{3}$

Powers and Roots

FOUNDATION

HIGHER

Calculations Using Index Laws

The objective is for students to understand and use the index laws. Students should be able to use the index laws for multiplication, division and raising to a power in order to perform calculations involving indices. For example, students should be able to simplify $(7^3 \times 7^5)^2$, writing the result as a single power of 7.

Powers and Roots

FOUNDATION

HIGHER

Calculations with Fractional Indices

The objective is for students to be able to work with fractional indices. Students should know that the denominator of a fractional power is equivalent to a root. Students should be able to evaluate numbers with fractional indices. Students should also be able to perform calculations with numbers that include fractional indices. For example, students should be able to work out $8^{(-2/3)}$

Powers and Roots

HIGHER

{Estimate powers and roots of any given positive number}

NUMBER

1

Estimate Powers and Roots of a Given Positive Number

The objective is for students to estimate powers and roots of a positive number. Students should be able to identify between which two integers the square or cube root of a positive number lies in order to estimate the root. They should also be able to use powers of integers to estimate powers of rational numbers. For example: \bullet Estimate the square root of 110 \bullet Estimate 3.2^3

Powers and Roots

HIGHER

Calculate with numbers in standard form $A \times 10^n$, where $1 \leq A < 10$ and n is an integer

NUMBER

3

Standard Form

Powers and Roots

The objective is to understand and use standard form. Students should be able to recognise numbers that are in standard form, convert numbers between standard and non-standard form and order numbers written in standard form. Students should also become comfortable interpreting their calculator display for standard form. For example: • Express 370000 in standard form. • Express 3.7×10^{-3} as an ordinary number • Write the following in ascending order 1.34×10^3 , 13.4×10^2 , 135

FOUNDATION

HIGHER

Multiplying and Dividing Numbers in Standard Form

Powers and Roots

The objective is to multiply and divide numbers in standard index form. Students should be able to multiply and divide numbers in standard index form. For example, calculating that $(3 \times 10^7) \times (2 \times 10^5) = 6 \times 10^{12}$

FOUNDATION

HIGHER

Adding and Subtracting Numbers in Standard Form

Powers and Roots

The objective is to add and subtract numbers in standard index form by writing them as equal powers of ten. Students should be able to add and subtract numbers in standard index form for example $3 \times 10^7 + 2 \times 10^5 = 3 \times 10^7 + 0.02 \times 10^7 = 3.02 \times 10^7$

FOUNDATION

HIGHER

Calculate exactly with fractions, {surds} and multiples of π ; {simplify surd expressions involving squares [for example $\sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$] and rationalise denominators}

NUMBER

6

Calculate with Fractions

Fractions

The objective is to perform arithmetic calculations with fractions. Students should be able to: • identify equivalent fractions •simplify fractions •convert between mixed numbers and improper fractions •compare fractions • add, subtract, multiply and divide fractions. For example: • Write the following fractions in ascending order $\frac{1}{2}$, $\frac{1}{4}$, $\frac{2}{3}$, $\frac{1}{8}$ • Calculate $\frac{3}{5} + \frac{2}{3}$ •Calculate $1\frac{2}{3} * \frac{3}{4}$

FOUNDATION

HIGHER

Solving Problems involving Fractions

Fractions

The objective is for students to use their skills of calculating exactly with fractions to solve problems. Students should be able to use fractions in a variety of contexts. For example, Alicia is cycling a distance of $9\frac{3}{4}$ km. After $6\frac{1}{2}$ km she decides to walk the rest of the way. How far does she walk?

FOUNDATION

HIGHER

Calculate with Multiples of π

Properties of Numbers

The objective is to perform calculations with multiples of π . Students should be able to find the circumference, area, radius or diameter of a circle, leaving their result in terms of π . For example, students can express the area of a circle with a radius of 4cm as 16π cm².

FOUNDATION

HIGHER

Simplifying Surds

Powers and Roots

The objective is for students to use their knowledge of calculations with roots to simplify surds. Students should be able to write integers as products of prime factors in order to simplify surds. For example, students can simplify $\sqrt{32}$ to $4\sqrt{2}$

HIGHER

Calculating with Surds

Powers and Roots

The objective is for students to calculate exactly with surds. Students should be able to simplify expressions using the rules of surds and perform calculations involving addition, subtraction, multiplication and division of surds. For example: • Simplify fully $\sqrt{50} * \sqrt{2}$ • Show that $(\sqrt{30}+3)(\sqrt{3}-\sqrt{10})$ can be simplified to $a\sqrt{3}$ where a is an integer

HIGHER

Rationalising Denominators

Powers and Roots

The objective is for students to rationalise the denominator of a fraction. Students should be able to rationalise the denominator when it has one or more terms. For example: •Write $\frac{3}{\sqrt{5}}$ as $\frac{3\sqrt{5}}{5}$ • Write $\frac{1}{(4+\sqrt{5})}$ as $\frac{(4-\sqrt{5})}{11}$ •Show that $(\frac{8}{(2\sqrt{3})})-(\frac{4\sqrt{3}}{5})$ simplifies to $(\frac{8\sqrt{3}}{15})$

HIGHER

Simplify and manipulate algebraic expressions (including those involving surds {and algebraic fractions}) by simplifying expressions involving sums, products and powers, including the laws of indices

ALGEBRA

6

Manipulating Expressions

The objective is for students to re-write expressions in different forms. Students should be able to expand a term over a single bracket, simplify indices and collect like terms. For example, students can simplify: $\bullet 3x - 2 + 4(x + 5)$ $\bullet 2x^2(3x - y)$ $\bullet 6x - 4x^4(5 - 2x^{-3})$ $\bullet x^{16} \div x^4$

Using Equations and Functions

FOUNDATION

HIGHER

Factorising Algebraic Expressions

The objective is for students to appreciate that factorising allows expressions to be re-written as products rather than polynomials. Students should be able to factorise algebraic expressions by taking out common factors. For example, students can factorise: $\bullet 6x^2 + 2y$ $\bullet 3x^2y - 9y$ $\bullet 4x^3y^5 + 2xy^3$

Using Equations and Functions

FOUNDATION

HIGHER

Manipulating Binomial Expressions

The objective is for students to multiply binomial terms of the form $(ax \pm b)$. Students should be able to multiply 2 binomial expressions. Higher students should be able to multiply more than 2 binomial expressions and those including indices. For example, students can multiply out and simplify $\bullet (x + 5)(x - 2)$ $\bullet (4x + 5)(3x - 2)$ \bullet (Higher) $(2x^3 - 1)(5x - 2)$ \bullet (Higher) $(x + 2)^3$

Using Equations and Functions

FOUNDATION

HIGHER

Factorising Quadratic Expressions

The objective is for students to re-write quadratic expressions of the form $x^2 + bx + c$ as products by factorising. Students should be able to articulate the link is between the sum and product of 'c' and 'd' when $(x + c)(x + d)$ are multiplied out. For example, students can factorise $\bullet x^2 + 7x + 12$ $\bullet x^2 - x - 12$ $\bullet x^2 - 7x + 12$

Using Equations and Functions

FOUNDATION

HIGHER

Factorise Quadratic Expressions

The objective is for students to extend their knowledge of factorising quadratic expressions. Students should be able to recognise the difference of two squares of the form $x^2 - a^2$ Higher students should be able to factorise quadratics of the form $ax^2 + bx + c$. For example: \bullet Factorise $x^2 - 9$ \bullet (Higher) Factorise $9x^2 - 4$ \bullet (Higher) Factorise $2x^2 + 11x + 12$

Using Equations and Functions

FOUNDATION

HIGHER

Algebraic Fractions

The objective is for students to apply their skills of factorisation and multiplying binomials to simplifying algebraic fractions. This includes cancelling common factors and adding/simplifying fractions. Higher students should be able to simplify a fraction with a quadratic expression in the numerator and denominator. For example, students should be able to simplify $\bullet 25x/5$ \bullet (Higher) $(4x+6)/(6x^2+19x+15)$

Using Equations and Functions

FOUNDATION

HIGHER

Translate simple situations or procedures into algebraic expressions or formulae; derive an equation, solve the equation and interpret the solution

ALGEBRA

2

Writing Expressions and Equations

Using Equations and Functions

The objective is for students to write expressions and equations in order to solve problems in context. Students should be able to set up, rearrange and solve simple linear equations. For example: Three siblings have the following age relationships: the oldest is three times the youngest; the middle is three years younger than the oldest; the sum of all three ages is 33 years. Write down expressions for the three ages and form an equation. Solve this equation to find the age of the youngest sibling.

FOUNDATION

HIGHER

Using and Rearranging Formulae

Using Equations and Functions

The objective is for students to write and use formulae. Students should be able to form, use, substitute into, and re-arrange formulae. For example, students should be able to construct a formula to give earnings based on an hourly rate, hours worked and a fixed bonus. They should be able to re-arrange their formula to make the 'number of hours' the subject.

FOUNDATION

HIGHER

Use the form $y = mx + c$ to identify parallel {and perpendicular} lines; find the equation of the line through two given points, or through one point with a given gradient

ALGEBRA

3

Finding the Equation of a Line

Using Equations,
Functions and Graphs

The objective is for students to build on their previous knowledge of using $y = mx + c$. Students should be able to:

- Calculate the gradient of a straight-line given two points
- Work out the equation of a line, given two points on the line. For example, students should be able to write the equation of the line given the points (-5, 1) and (10, 10).

FOUNDATION

HIGHER

Finding the Equation of a Line

Using Equations,
Functions and Graphs

The objective is for students to build on their previous knowledge of using $y = mx + c$ to find the equation of a line given one point and the gradient. Students should be able to substitute and manipulate $y = mx + c$ or use $y - y_1 = m(x - x_1)$. For example, students can write the equation of the line in the form $y = mx + c$ given the gradient of $2/3$ and the point (6,8)

FOUNDATION

HIGHER

Equations of Parallel & Perpendicular Lines

Using Equations,
Functions and Graphs

The objective is for students to recognise and understand the properties of parallel and perpendicular lines on a coordinate grid. Students should be able to use their skills of substitution and manipulation on the form $y = mx + c$ to tell whether lines are parallel or not. Higher students will also be able to work out the gradients of lines that are parallel and perpendicular to a given line and show that two lines are parallel or perpendicular using gradients. For example:

- Show that the lines $2x = 3y - 12$ and $6y = 4x - 8$ are parallel.
- (Higher) Show that the points (0,4) (2,3) (3,10) and (5,9) form a rectangle.

FOUNDATION

HIGHER

Translate simple situations or procedures into two simultaneous equations, solve the equation(s) and interpret the solution

ALGEBRA

3

Solving Simultaneous Linear Equations

The objective is for students to solve simultaneous linear equations. Students should appreciate the difference between linear equations that have one variable and those that contain two variables and the link with the number of solutions. Students should be able to solve pairs of equations using substitution or simple elimination. For example, students can find the solutions to $x+3y=10$ and $x-y=2$.

Using Equations,
Functions and Graphs

FOUNDATION

HIGHER

Solving Simultaneous Linear Equations

The objective builds on students' understanding of simultaneous linear equations to solve increasingly complex pairs of equations using substitution or elimination. Students should be able to solve pairs of linear equations where the coefficients are not the same on either variable. For example, students can solve pairs of equations such as: $2x + 3y = 11$ and $5x - 2y = 18$

Using Equations,
Functions and Graphs

FOUNDATION

HIGHER

Forming and Solving Simultaneous Linear Equations

The objective builds on students' skills of writing linear equations and solving simultaneous equations in order to now write and solve simultaneous equations for situations in context. Students should be able to translate problems into a pair of simultaneous linear equations, solve them and interpret the solutions. For example: It costs £23 to download 4 tunes and 5 Apps. It costs £20 to download 1 tune and 6 Apps. How much does 1 App cost?

Using Equations,
Functions and Graphs

FOUNDATION

HIGHER

Solve two simultaneous equations in two variables (linear/linear) algebraically; find approximate solutions using a graph

ALGEBRA

1

Simultaneous Linear Equations - Graphical Solutions

The objective is for students to find approximate solutions to simultaneous linear equations by using the point of intersection of two straight lines. Students should be able to link types of solutions to graphical representations: unique, infinite and impossible. For example, by plotting suitable values for the equations $4x - y = -1$ and $x + y = 5$, students should be able to find solutions in the region of $x = 0.8$ and $y = 4.2$

Using Equations,
Functions and Graphs

FOUNDATION

HIGHER

{Find approximate solutions to equations numerically using iteration}

ALGEBRA

2

Trial and Improvement

Using Equations and Functions

The objective is for students to understand that where equations are not easily solved, a numerical method may be appropriate. Students should be able to use systematic trial and improvement to find approximate solutions of equations to a given degree of accuracy. For example, given that there is a solution between 5 and 6, they should be able to use trial and improvement to solve the equation $2x^3 - 3x = 400$ to 1 decimal place.

HIGHER

Recursive Formulae

Using Equations and Functions

The objective is for students to work with recursive formulae. Students should be able to use suffix notation in recursive formulae and use such formulae to find approximate solutions. For example, using an initial value of $x_1 = 1$, students should be able to use the recursive formula $x_{n+1} = 3 - 1/x_n$ to solve the equation $x = 3 - 1/x$ to 1 decimal place.

HIGHER

Where appropriate, interpret simple expressions as functions with inputs and outputs; {interpret the reverse process as the 'inverse function'; interpret the succession of two functions as a 'composite function'}

ALGEBRA

4

Number Machines

Using Equations and Functions

The objective is to consolidate and build upon students' abilities in representing functions as number (function) machines. Students should be able to use and understand number machines and be comfortable in drawing them to represent an expression. They should also be able to form expressions when given number machines. For example: • Complete a number machine to represent the function $y = 3x - 1$ • Write down the function shown by a number machine $x \rightarrow *8 \rightarrow +3 \rightarrow y$

FOUNDATION

HIGHER

Number Machines and Inverses

Using Equations and Functions

The objective is for students to apply their knowledge of number machines and inverses to solve problems. Students should be able to work backwards using number machines to perform inverse operations and find values. Higher students should be able to articulate this process as the "inverse function." For example, use the function machine $x \rightarrow *4 \rightarrow +3 \rightarrow 23$, to find the value of x .

FOUNDATION

HIGHER

Using Function Notation

Using Equations and Functions

The objective introduces students to the notion of function notation, $f(x)$. Students should be able to understand that a function is a relationship between two sets of values. They should be comfortable in using the correct mathematical notation to represent a function, substituting values into functions and solving equations that use function notation. For example: • Given that $f(x) = 5x + 4$, find the value of $f(4)$ • Given that $f(x) = 2x - 6$, solve $f(x) = 0$.

HIGHER

Composite and Inverse Function Notation

Using Equations and Functions

The objective introduces students to more complex notation associated with composite and inverse functions. Students should be able to understand the concept of function notation, use the notation to find the value of composite functions and write expressions for inverse functions. For example: • $f(x) = 3 + x$ and $g(x) = x^2$, find the value of $fg(2)$ • $f(x) = 3x - 5$, work out an expression for $f^{-1}(x)$.

HIGHER

Know the exact values of $\sin\theta$ and $\cos\theta$ for $\theta = 0^\circ, 30^\circ, 45^\circ, 60^\circ$ and 90° ; know the exact value of $\tan\theta$ for $\theta = 0^\circ, 30^\circ, 45^\circ$ and 60°

GEOMETRY AND MEASURES

1

Special Trig Angles

Geometry

The objective is to build on the students' knowledge of special triangles to identify the exact values of \sin , \cos and \tan for $0, 30, 45$ and 60 degrees. Students should be able to recall the exact values and should then be able to answer questions involving these angles, leaving their answers in an exact form, without the need of a calculator. For example calculating the missing sides in a right-angled triangle when given one side and one of the special angles.

FOUNDATION

HIGHER

Recognise, sketch and interpret graphs of linear functions, quadratic functions, simple cubic functions, the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$, {the exponential function $y = kx$ for positive values of k , and the trigonometric functions (with arguments in degrees) $y = \sin x$, $y = \cos x$ and $y = \tan x$ for angles of any size}

ALGEBRA

3

Sketching Quadratic and Cubic Functions

Using Equations,
Functions and Graphs

The objective is for students to be able to recognise and sketch the shape of quadratic graphs and cubic graphs. They should be able to create a table of values and plot accurate graphs of cubic and quadratic functions. Additionally, students should be able to calculate values for a quadratic and draw the graph. Students should also be able to recognise simple translations of the graphs $y = x^2$ and $y = x^3$. Students should also be able to find a value of x for a given value of y using the graph. For example, students should be able to sketch the graph of $y = x^3 + 4$ and use the graph to find y when $x = 2$.

FOUNDATION

HIGHER

Sketching Reciprocal and Exponential Graphs

Using Equations,
Functions and Graphs

The objective is for students to be able to recognise and sketch reciprocal graphs. Higher students should be able to recognise and sketch exponential graphs of the form $y = k^x$ for positive values of k . They should be able to create a table of values, plot and interpret accurate graphs. Students should also be able to find a value of x for a given value of y using the graph. For example, students should be able to sketch the graph of $y = \frac{1}{2x}$ and use the graph to find y when $x = 4$.

FOUNDATION

HIGHER

Sketching Trigonometric Graphs

Using Equations,
Functions and Graphs

The objective is for students to be able to recognise and sketch the shape of sine, cosine and tangent graphs. Students should be able to complete a table of values and plot the graphs of sine, cosine and tangent. Students should understand the features of the graphs, such as periodicity, local maximums and minimums. Students should be able to identify the solutions of simple trig equations between 0 and 360 using symmetry. For example, students can draw a graph of $y = \cos(x)$ from 0 to 180 degrees and use it to find solutions to $\cos(x) = -0.5$ between 0 and 360 degrees using symmetry.

HIGHER

Plot and interpret graphs (including reciprocal graphs {and exponential graphs}) and graphs of non-standard functions in real contexts, to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration

ALGEBRA

2

Plotting Real Life
Reciprocal and
Exponential Graphs

Using Equations,
Functions and Graphs

The objective is for students to plot and interpret graphs of reciprocals and non-standard functions in context. Higher students will also be able to plot and interpret exponential graphs in context. Students should be able to plot a graph representing a real life problem from a given situation, a table of values or a formula. They should be able to interpret these graphs and answer questions concerning the graph. For example: • Given a fixed charge and cost per day to hire a car, students can plot a graph to work out the cost of hiring the car for a given amount of days • (Higher) Given a principal balance and compound interest rate, students can plot the graph and find the balance in the account after a given time period.

FOUNDATION

HIGHER

Plotting Real Life
Kinematic Graphs

Using Equations,
Functions and Graphs

The objective is for students to plot and interpret graphs representing Kinematic problems involving distance, speed time. Students should be able to plot a graph representing a real life problem from a given situation, a table of values or a formula. They should be able to interpret these graphs and answer questions concerning the graph. For example, students can draw distance-time graph for a car journey and find the average speed of the journey.

FOUNDATION

HIGHER

{Sketch translations and reflections of the graph of a given function}

ALGEBRA

3

Translating Functions

Using Equations,
Functions and Graphs

The objective is for students to sketch the image of a function after a translation in the y-direction, $f(x)+a$, and in the x-direction, $f(x+b)$. They should be able to understand the translation in function notation and perform the translation when the instruction is given in both written and vector form. Students should also be able to recognise a translation and be able to write down the function of a transformation given the original function. For example: • Translate the function of $y = 2x + 4$ three places left. • Translate the function $y = 3x - 4$ by a given vector. • Given the graph of $y=x^3$, students can sketch the graph of $y=x^3+4$. • Given the graph of $y=f(x)$ and the graph of a transformation of $y=f(x)$, students can write down the equation of the transformed function.

HIGHER

Reflecting Functions

Using Equations,
Functions and Graphs

The objective is for students to sketch images of functions after a reflection. They should be able to understand the reflection in function notation, $-f(x)$ and $f(-x)$ and reflect functions in the y-axis and x-axis when given the appropriate instruction. Students should also be able to recognise a reflection and be able to write down the function of a transformation given the original function. For example: • Reflect the function $y = 2x + 4$ in the y-axis. • Given the graph of $y=\cos(x)$, students can sketch the graph of $y=-\sin(x)$. • Given the graph of $y=f(x)$ and the graph of a transformation of $y=f(x)$, students can write down the equation of the transformed function.

HIGHER

Translating and Reflecting Functions

Using Equations,
Functions and Graphs

The objective is for students to build upon previous experiences of sketching single transformations by combining transformations. Students should be able to sketch the image of a function when asked to perform two transformations. For example, sketch the image of the function of $y = 4x - 5$ after a reflection in the x-axis, followed by a translation 3 places down.

HIGHER

Identify and apply circle definitions and properties, including: centre, radius, chord, diameter, circumference, tangent, arc, sector and segment

GEOMETRY AND MEASURES

1

Circle Definitions and Properties

The objective is to recap and instil confidence in the vocabulary and properties of circles. Students should be able to recall the definition of a circle; identify, name and draw parts of the circle; and draw a circle given the radius or diameter. For example, students can draw a circle of radius 4 cm and draw and label a chord.

FOUNDATION

HIGHER

Properties of Shapes

{Apply and prove the standard circle theorems concerning angles, radii, tangents and chords, and use them to prove related results}

GEOMETRY AND MEASURES

4

Circle Theorems: Angles

Geometry

The objective is for students to apply and prove the standard circle theorems concerning angles. Students should be able to recall and use:

- The angle subtended by an arc at the centre of a circle is twice the angle subtended at any point on the circumference.
- Angles in the same segment are equal.
- An angle subtended at the circumference by a semicircle is a right angle.

For example, students can work out angles in an arc given an angle in the same arc.

HIGHER

Circle Theorems: Angles & Chords

Geometry

The objective is for students to apply and prove the standard circle theorems concerning chords. Students should be able to use congruent triangles to explain why the perpendicular from the centre to a chord bisects the chord. Students should understand that inscribed regular polygons can be constructed by equal division of a circle. Students should be able to prove and use the fact that opposite angles of a cyclic quadrilateral sum to 180 degrees. For example, given a cyclic quadrilateral and 2 angles, students can work out the missing angles and give reasons for their answers.

HIGHER

Circle Theorems: Tangents

Geometry

The objective is for students to understand and apply the standard circle theorems concerning tangents. Students should be able to understand and use the fact that:

- The tangent at any point on a circle is perpendicular to the radius at that point.
- Tangents from an external point are equal in length.

Students should be able to prove and use the alternate segment theorem. For example, given a circle, tangents, and an angle of length, students can find a missing angle or length and give reasons.

HIGHER

Circle Theorems

Geometry

The objective is to consolidate students' understanding of the standard circle theorems. Students should be able to apply the standard circle theorems concerning angles, radii, tangents and chords to prove related results. For example, given a triangle drawn inside a circle and the angle on the arc, students can prove whether the opposite side is a diameter.

HIGHER

{Recognise and use the equation of a circle with centre at the origin; find the equation of a tangent to a circle at a given point}

GEOMETRY AND MEASURES

2

Equation of a Circle

Properties of Shapes

The objective is for students to be able to write and use the equation of a circle. Students should be able to recognise and write down the equation of a circle with centre (0,0) and radius r . They should also be able to find the points of intersection of a given circle and straight line. For example: • Students can find the centre and exact value of the radius of the circle given the equation $x^2+y^2=3$. • Given the centre (0,0) and radius 8, students can write the equation of the circle. • Given the equation of a circle $x^2+y^2=5$ and the line $y=x-1$, students can find the points of intersection and the length of the chord between them.

HIGHER

Finding the Equation of the Tangent

Properties of Shapes

The objective is for students to find the equation of the tangent at a given point. Students should be able to use the fact that the radius is perpendicular to the tangent at a given point in order to work out the gradient of the tangent and use that to find the equation of the tangent. For example, given the point (2,2) and the equation $x^2+y^2=8$, students can find the equation of the tangent at point (2,2)

HIGHER

Apply Pythagoras' Theorem and trigonometric ratios to find angles and lengths in right-angled triangles {and, where possible, general triangles} in two {and three} dimensional figures

GEOMETRY AND MEASURES

6

Pythagoras' Theorem

Geometry

The objective is to instil confidence in understanding and using Pythagoras' theorem in 2D problems. Higher students will build on their skills of Pythagoras' theorem in 2D and be able to use Pythagoras' theorem in 3D problems. Students should be able to find missing lengths of given right angled triangles. For example: •Given two sides of a right triangle, students can work out the missing side.

FOUNDATION

HIGHER

Sine Ratio

Geometry

The objective is to instil confidence in understanding and using the sine ratio in 2D problems. Students should be able to use the sine ratio to find missing lengths or missing angles in right-angled triangles. For example: •Students can use the sine ratio to find a missing angle given the length of the hypotenuse and opposite side. •Given an angle and the length of either the hypotenuse or opposite side, students can find the missing side.

FOUNDATION

HIGHER

Cosine Ratio

Geometry

The objective is to instil confidence in understanding and using the cosine ratio in 2D problems. Students should be able to use the cosine ratio to find missing lengths of missing angles in right-angled triangles. For example: •Students can use the cosine ratio to find a missing angle given the length of the hypotenuse and adjacent side. •Given an angle and the length of either the hypotenuse or adjacent side, students can find the missing side.

FOUNDATION

HIGHER

Tangent Ratio

Geometry

The objective is to instil confidence in understanding and using the tangent ratio in 2D problems. Students should be able to use the tangent ratio to find missing lengths and missing angles in right-angled triangles. For example: •Students can use the tangent ratio to find a missing angle given the length of the opposite and adjacent sides. •Given an angle and the length of either the opposite or adjacent side, students can find the missing side.

FOUNDATION

HIGHER

Solving Problems Involving Pythagoras' Theorem and the Trigonometric Ratios

Geometry

The objective is to combine students' skills in using Pythagoras' Theorem and Trigonometry in order to solve problems in 2D. Students should be able to identify when to use Pythagoras' Theorem and which trigonometric ratio(s) to use in order to solve problems. For example: • Given the height of a building and the angle between the ladder and the ground, students can work out the length of a ladder needed to reach the top of the building •(Higher) Given a diagram of two right angled triangles joined by one common side, students can work out a missing length and find the area of the whole shape.

FOUNDATION

HIGHER

Solving Problems Involving Pythagoras' Theorem and the Trigonometric Ratios

Geometry

The objective is to combine students' skills in using Pythagoras' Theorem and Trigonometry in order to solve problems in 2D, including those involving bearings. Students should be able to identify when to use Pythagoras' Theorem and which trigonometric ratio(s) to use. Higher students should build on these skills in order to solve problems in 3D, including finding the angles between a line and a plane. For example: • Given a bearing that results in a right-angled triangle and distances travelled students can calculate the distance between 2 points •(Higher) Given a cuboid, students can work out the length of a diagonal between a vertex at the base and at the top •(Higher) Given a square based pyramid and 3 sides, students can work out the height and a missing angle.

FOUNDATION

HIGHER

{Know and apply the sine rule, $a / \sin A = b / \sin B = c / \sin C$, and cosine rule, $a^2 = b^2 + c^2 - 2bc \cos A$, to find unknown lengths and angles}

GEOMETRY AND MEASURES

4

The Sine Rule

Geometry

The objective is for students to know and apply the sine rule. Students should be able to find missing lengths and angles. For example: • Given 2 angles of a scalene triangle and the length of one opposite side, students can find the lengths of the other opposite side. • Given the length of 2 sides of a scalene triangle and an opposite angle, students can find the other opposite angle

HIGHER

The Cosine Rule

Geometry

The objective is for students to know and apply the cosine rule. Students should be able to find missing lengths and angles. For example: • Given the lengths of all 3 sides of a scalene triangle, students can find the angles between them. • Given 2 adjacent sides and the angle between them, students can find the missing side length.

HIGHER

Solving Problems Involving the Sine and Cosine Rule

Geometry

The objective is for students to use the sine and cosine rules to solve problems. Students should be able to combine their knowledge of the sine and cosine rules to find missing lengths and angles. For example, given the lengths two adjacent sides an opposite angle, students can find the missing side length.

HIGHER

Solving Problems Involving the Sine and Cosine Rule

Geometry

The objective is for students to use the sine and cosine rule to solve problems. Students should be able to combine their knowledge of the sine and cosine rules to find missing lengths and angles. For example, given distances and bearings, students can calculate the distance and bearing between two points.

HIGHER

{Know and apply Area = $\frac{1}{2} ab \sin C$ to calculate the area, sides or angles of any triangle}

GEOMETRY AND MEASURES

2

Finding the Area of a Triangle using Trigonometry

The objective is for students to know and apply the area formula $\text{Area} = \frac{1}{2} ab \sin C$ to calculate the area of a triangle. Students should be able to calculate the area of a triangle given the length of 2 sides and the angle between them. For example, given a scalene triangle with side lengths of 3cm and 4cm and angle of 35 degrees between them, students can work out the area of the triangle.

HIGHER

Geometry

Solving Problems Involving the Area of a Triangle using Trigonometry

The objective is to build on students knowledge of the area formula $\text{Area} = \frac{1}{2} ab \sin C$ to solve problems. For example, students should be able to work out the area of a parallelogram given the base, a slant height and the angle between them.

HIGHER

Geometry

Compare lengths, areas and volumes using ratio notation and/or scale factors making links to similarity including trigonometric ratios

GEOMETRY AND MEASURES

2

The Trigonometric Ratios in Similar Shapes

The objective is for students to make links to similarity using trigonometric ratios. Students should be able to apply their understanding of similarity and the trigonometric ratios to solve problems. For example, given two right-angle triangles with a common angle, students can find the unknown sides.

FOUNDATION

HIGHER

Geometry

The Trigonometric Ratios in Similar Shapes

The objective is for students to make links to similarity using trigonometric ratios. Students should be able to apply their understanding of similarity and the trigonometric ratios to solve problems. For example, given a right-angled triangle, the value of the tangent ratio of an angle and the length of the opposite side, students can calculate the length of the side adjacent to the angle and use that to calculate a missing side length of an enlarged triangle.

FOUNDATION

HIGHER

Geometry

Interpret and use bearings

Bearings

Position and Direction

The objective is for students to interpret and use bearings. Students should be able to use three-figure bearings to specify direction, mark and work out points and bearings on scale drawings and diagrams given the bearing from another point. For example: •Given the bearing to B from A, students can work out the bearing to A from B. •Use accurate constructions to locate a point on a map or scale drawing.

HIGHER

Convert between common units of measure

GEOMETRY AND MEASURES

1

Converting Measures

Measures

The objective is for students to convert between common units of measure. Students should be able to convert between metric measures for length, area, volume and capacity; time; and between metric and imperial units

FOUNDATION

HIGHER

Convert between related compound units (speed, rates of pay, prices, density, pressure) in numerical and algebraic contexts

GEOMETRY AND MEASURES

2

Converting Compound Measures

Measures

The objective is for students to convert between related compound units in numerical contexts. Students should be able to convert between compound measures involving metric and imperial units. For example, given a speed in km/h, students can calculate the speed in mph.

FOUNDATION

HIGHER

Converting Compound Measures in Context

Measures

The objective is for students to convert compound measures. Students should be able to convert between related compound units in algebraic contexts. For example, given a situation in context of distance between two places in km and time taken in minutes, students can calculate the speed in mph.

FOUNDATION

HIGHER

Use dimensions to identify whether a formula represents a length an area or a distance and derive a formula for a compound measure from its units

GEOMETRY AND MEASURES

Dimensional Analysis

Measures

The objective is for students to use dimensions to identify whether a formula represents a length, area, or volume. Students should recognise whether a formula will result in linear, squared, or cubed dimensions. For example, given that a, b, and c are lengths, students can explain why abc can represent a volume.

FOUNDATION

HIGHER

Deriving Formulae for Compound Measures

Measures

The objective builds on students' skills of dimensional analysis to derive formulae for a compound measure from its units. Students should be able to work backwards from the units to derive a formulae. For example, given a density in g/m^3 , students can derive that the formula for density is mass/volume.

FOUNDATION

HIGHER

Solve quadratic equations {including those that require rearrangement} algebraically by factorising, {by completing the square and by using the quadratic formula}; find approximate solutions using a graph

ALGEBRA

Solving Quadratic Equations by Factorising

Using Equations and Functions

The objective builds on students' skills of factorising quadratic expressions in order to now solve quadratic equations by factorising. Students should be able to recognise a quadratic equation and factorise it to solve. Higher students should be able to solve quadratic equations of the form ax^2+bx+c . For example: • Solve $x^2+7x+12=0$. •(Higher) Solve $3x^2-10x-8$

FOUNDATION

HIGHER

Solving Quadratic Equations by Factorising

Using Equations and Functions

The objective builds on students' skills of solving quadratic equations by factorising to now solve quadratic equations that require rearrangement by factorising. Students should be able to recognise a quadratic equation, rearrange it and use factorisation to solve it. Higher students should be able to solve quadratic equations of the form ax^2+bx+c . For example, students can solve $x^2+10=7x$.

FOUNDATION

HIGHER

Approximating Solutions to Quadratic Equations by Graphing

Using Equations, Functions and Graphs

The objective builds on students' skills of graphing quadratic equations in order to approximate solutions. Students should be able to approximate roots from a graph in order to solve equations of the type $0=ax^2+bx+c$. For example, students can solve $2x^2+2x-4=0$ by either plotting or being given the graph and finding the roots.

FOUNDATION

HIGHER

Solving Quadratic Equations by Completing the Square

Using Equations, Functions and Graphs

The objective is for students to solve quadratic equations of the form $x^2+bx+c=0$ by completing the square. Students should be able to understand the process of completing the square, then use algebra to solve quadratic equations by completing the square. For example, students can solve $x^2-4x-3=1$ by completing the square.

HIGHER

Solving Quadratic Equations using the Quadratic Formula

Using Equations, Functions and Graphs

The objective is for students to solve quadratic equations using the quadratic formula. Students should be able to identify the values of a, b, and c from a quadratic equation, rearranging where necessary, and substitute those values into the quadratic formula in order to find solutions to the equation. Students should also be able to solve an algebraic equation with linear expressions in the denominators by manipulating the equation until it is of the form $ax^2+bx+c=0$ and then use the quadratic formula to solve it. For example, students can solve $4x^2-3x-2=8$ by using the quadratic formula.

HIGHER

Solve quadratic equations {including those that require rearrangement} algebraically by factorising, {by completing the square and by using the quadratic formula}; find approximate solutions using a graph

ALGEBRA

Solving Problems involving Quadratic Equations in Context

Using Equations, Functions and Graphs

The objective is for students to be able to solve quadratic equations in context, including geometry problems that lead to a quadratic equation. Students should be able to write and solve quadratic equations of various situations. Higher students should be able to write equations of the form $ax^2+bx+c=0$ and solve by factorising, completing the square, or using the quadratic formula as appropriate. For example, given the perimeter of a rectangle and side lengths in x of a rectangle, students can work out the area of the rectangle.

FOUNDATION

HIGHER

Identify and interpret roots, intercepts and turning points of quadratic functions graphically; deduce roots algebraically {and turning points by completing the square}

ALGEBRA

Y-intercept of a Quadratic Function

Using Equations, Functions and Graphs

The objective is for students to understand that the constant term in a quadratic function is the y-intercept. Students should understand and be able to use this idea to interpret the y-intercept of a given graph. For example, given a graph representing parabolic motion, students can identify the starting height.

FOUNDATION

HIGHER

Roots of a Quadratic Function

Using Equations, Functions and Graphs

The objective is for students to understand that the roots of a quadratic function are the x-intercepts. Students should understand and be able to use the idea that the roots are the solution to the equation $ax^2+bx+c=0$ and interpret the roots given a graph. For example, given the graph representing the area of a rectangle with sides lengths in terms of x , students can identify the roots and thus the side lengths of the rectangle.

FOUNDATION

HIGHER

Finding Roots of a Quadratic Function Algebraically

Using Equations, Functions and Graphs

The objective is for students to understand that the roots of a quadratic function are the solutions to $f(x)=0$. Students should be able to find the x-intercepts by finding the x-values of a quadratic function when $y=0$. For example, students can find the x-intercepts of the function $y=x^2+7x+10$ by factorising to solve $x^2+7x+10=0$.

FOUNDATION

HIGHER

Identify and interpret roots, intercepts and turning points of quadratic functions graphically; deduce roots algebraically {and turning points by completing the square}

ALGEBRA

Turning Points of a Quadratic Function

The objective is for students to understand that the turning point of a quadratic function represents the maximum or minimum value that the function can take. Students should be able to use this idea to interpret the turning point of a given graph. For example, given a graph of parabolic motion, students can identify the maximum height reached.

Using Equations,
Functions and Graphs

FOUNDATION

HIGHER

Finding Turning Points of a Quadratic Function Algebraically

The objective is for students to understand that completing the square will give the turning points of a quadratic function. Students should be able to complete the square of a given function and explain how completing the square gives the coordinates of the maximum or minimum. For example, given the function $y=x^2+8x+13$ students can complete the square to find the turning point.

Using Equations,
Functions and Graphs

HIGHER

Sketching Quadratic Graphs

The objective builds on students' skills in finding the y-intercept, roots, (and turning points for higher students) of a quadratic function algebraically in order to now sketch graphs of quadratic functions. Students should be able to find the y-intercept, roots and turning point given a quadratic function and sketch it. For example, given the function $y=x^2+x-2$, students can identify the y-intercept, find the roots by factorising and find the minimum by completing the square. Students can then sketch a graph of the function by plotting these points and using symmetry.

Using Equations,
Functions and Graphs

FOUNDATION

HIGHER

Solve two simultaneous equations in two variables (quadratic) algebraically; find approximate solutions using a graph

ALGEBRA

Approximating Solutions to Simultaneous Equations involving a Quadratic Graphically

Using Equations, Functions and Graphs

The objective is for students to find approximate solutions to two simultaneous equations where one is linear and the other quadratic given a graph. Students should understand that the intersection between the line and the parabola represent solutions to the simultaneous equations. Students should also be able to rearrange an equation in order to approximate solutions from a given graph. For example: • Given a graph of $y=x^2-x-2$ and $y=2$ students can approximate the solutions by reading the points of intersection off the graph. • Given a graph of $y=x^2-x-2$ students can approximate solutions to $5=x^2-x$ by comparing the equation to the expression x^2-x-2 and thus draw the line $y=7$ and finding the points of intersection.

HIGHER

Approximating Solutions to Simultaneous Equations involving a Quadratic Graphically

Using Equations, Functions and Graphs

The objective is for students to find approximate solutions to two simultaneous equations where one is linear and the other quadratic by graphing. Students should understand that the intersection between the line and the parabola represent solutions to the simultaneous equations. Students should be able to graph the parabola and the line in order to approximate solutions to the simultaneous equations. For example, students can graph the pair of simultaneous equations $y=x^2+x-6$ and $y=2x+1$ and find approximate solutions by reading the points of intersection off the graph.

HIGHER

Solving Problems involving Quadratic and Linear Simultaneous Equations Graphically

Using Equations, Functions and Graphs

The objective is for students to graphically solve two simultaneous equations where one is linear and the other quadratic in context. Students should be able to interpret or plot a graph representing a situation and use it to solve two simultaneous equations where one is linear and the other quadratic. For example, given a graph representing the area of a given rectangle with sides in terms of x , students can use it to find the area given various values of x by drawing the appropriate line on the graph and finding the points of intersection.

HIGHER

Solving Simultaneous Equations involving a Quadratic Algebraically

Using Equations, Functions and Graphs

The objective is for students to solve two simultaneous equations where one is linear and the other quadratic algebraically. Students should be able to rearrange and substitute the linear equation into the quadratic and solve the quadratic by factorising. For example, students can solve the pair of simultaneous equations $y=x^2-x-2$ and $y=x+1$ by substituting and factorising.

HIGHER

Solve two simultaneous equations in two variables (quadratic) algebraically; find approximate solutions using a graph

ALGEBRA

Solving Simultaneous Equations involving a Quadratic Algebraically

The objective is for students to solve two simultaneous equations where one is linear and the other quadratic algebraically. Students should be able to rearrange and substitute the linear equation in to the quadratic and solve by completing the square or using the quadratic formula. For example, students can solve the pair of simultaneous equations $y=x^2-x-2$ and $y=x-1$ by substituting and either completing the square or using the quadratic formula.

HIGHER

Using Equations,
Functions and Graphs

Solving Problems involving Quadratic and Linear Simultaneous Equations Algebraically

The objective is for students to algebraically solve two simultaneous equations where one is linear and the other quadratic in context. Students should be able to write and solve a pair of simultaneous equations where one is linear and one is quadratic. For example, given the net of a cylinder and dimensions of a rectangle from which the cylinder is cut, students can write and solve a pair of simultaneous equations linking the radius and height in order to find the possible volumes of the cylinder.

HIGHER

Using Equations,
Functions and Graphs

{Calculate or estimate gradients of graphs and areas under graphs (including quadratic and other non-linear graphs), and interpret results in cases such as distance-time graphs, velocity-time graphs and graphs in financial contexts}

ALGEBRA

Estimating the Gradient at a Point on Non-Linear Graphs

The objective is for students to estimate and use the gradient at a point on a curve. Students should be able to calculate an estimate for the gradient and interpret the meaning of the gradient at a point on a curve. For example, given a distance-time graph, students can estimate the gradient and interpret the gradient as the velocity.

HIGHER

Using Equations,
Functions and Graphs

Finding the Area under Non-Linear Graphs

The objective is for students to calculate the area under a graph. Students should be able to calculate the area under a graph consisting of straight lines and use the areas of trapezia, triangle and rectangles to estimate the area under a curve. For example, given a distance-time graph, students can calculate the area under various parts of the graph using known formulae for areas.

HIGHER

Using Equations,
Functions and Graphs

Solving Problems involving Gradients at a Point and Area under Graphs

The objective is for students to interpret the meaning of the area under a graph and the gradient at a point. Students should interpret the meaning of the product of the units of the variable on the vertical and horizontal axis. Students should make the connection between positive and negative gradients as increasing and decreasing rates. For example, given a velocity-time graph with velocity on the y-axis, students should be able to: • Estimate the acceleration at a given time by estimating the gradient • Calculate the distance by finding the area under the graph

HIGHER

Using Equations,
Functions and Graphs

{Interpret the gradient at a point on a curve as the instantaneous rate of change; apply the concepts of instantaneous and average rate of change (gradients of tangents and chords) in numerical, algebraic and graphical contexts}

NUMBER

Rates of Change

Ratio, Proportion and Rates of Change

The objective is for students to understand the meaning of the gradient as a rate of change. Students should be able to interpret the gradient as the rate of change of the variable on the vertical axis compared to the horizontal axis. Students should understand the difference between positive and negative gradients as rates of change. For example, given a distance-time graph, students can interpret the gradient as the speed, recognise relative increases or decreases in speed, and calculate an average speed of a journey.

HIGHER

Instantaneous Rate of Change

Ratio, Proportion and Rates of Change

The objective is for students to understand that the rate of change at a particular instant in time is represented by the gradient of the tangent to the curve at that point. Students should be able to draw a tangent at a point on a curve and measure its gradient. For example, given a graph of parabolic motion, students can find the speed of the object at a given time by drawing a tangent to the point and calculating its gradient.

HIGHER

Solving Problems involving Instantaneous Rate of Change

Ratio, Proportion and Rates of Change

The objective builds on students understanding of both average and instantaneous rates of change in order to solve problems involving these in context. Students should be able to interpret the meaning of gradients and calculate both an average and instantaneous rate of change where applicable. For example, given a distance-time graph, students can calculate and average speed of the journey as well as the speed at a particular time.

HIGHER

Set up, solve and interpret the answers in growth and decay problems, including compound interest {and work with general iterative processes}

NUMBER

Repeated Proportional Change

Ratio, Proportion and Rates of Change

The objective is to instil confidence in methods of calculating repeated proportional change. Student should be able to use a calculator to examine repeated proportional change. Higher students should be able to write formulae in order to calculate repeated proportional change. For example, given the original value and percentage of depreciation, students can calculate the value of a car after a given number of years.

FOUNDATION

HIGHER

Compound Interest

Fractions (Including Decimals And Percentages)

The objective is for students to set up, solve, and interpret problems involving compound interest. Students should be able to calculate balances and compare amounts. Higher students should be able to write formulae in order to calculate compound interest. For example, given an initial balance, one account with compound interest and another without, students can calculate the amount of interest in both accounts and compare them.

FOUNDATION

HIGHER

Growth and Decay

Fractions (Including Decimals And Percentages)

The objective is for students to set up, solve, and interpret growth and decay problems. Students should be able to calculate amounts at various times and interpret changes. Higher students should be able to articulate assumptions and limitations to such models. For example, students can calculate the amount of fish in a pond given a percentage growth in population.

FOUNDATION

HIGHER

Recognise and use quadratic sequences, and simple geometric progressions (r^n where n is an integer, and r is a positive rational number {or a surd}) {and other sequences}

ALGEBRA

Quadratic Sequences & nth Term Rule of the form an^2

The objective is for students to recognise quadratic sequences and to use nth term rules to solve problems. Students should be able to identify quadratic sequences by the second difference and find terms of a quadratic sequence given a rule. For example: • Is the following sequence quadratic: 5, 8, 13, 20, 29... ? • Find the first five terms of the sequence with nth term rule $3n^2+5n$. • The nth term rule of a sequence is $4n^2+5n$. Find the 6th term of the sequence and find the first term that is greater than 100.

FOUNDATION

HIGHER

Expressing Relationships

Geometric Progressions

The objective is for students to recognise and use geometric progressions of the form r^n where n is an integer and r is a positive rational number. Students should be able to recognise, use and write the nth term rule for a geometric progression. Higher students should be able to recognise and use geometric progressions when r is a surd. For example: • Find the first five terms of the sequence with nth term rule 3^n . • (Higher) Find the first five terms of the sequence with nth term rule $(\sqrt{8})^n$ and show that the fifth term is $128\sqrt{2}$.

FOUNDATION

HIGHER

Expressing Relationships

Further Geometric Progressions

The objective builds on students' understanding of geometric progressions to solve problems. Students should be able to write and use the nth term rule given a situation in context. For example: There are 1000 virus particles in a vial. The virus duplicates every hour. Write an expression representing the number of viruses after n hours. How many virus particles are there after 12 hours?

FOUNDATION

HIGHER

Expressing Relationships

Other Sequences

The objective is to introduce students to other sequences and nth term rules. Students should be able to find terms from nth term rules and write nth term rules for other sequences. For example: • Find the first five terms of the sequence with nth term rule $(3n+1)/n+2$. • Find the nth term rule of the sequence $1/5, 4/8, 9/11, 16/14, 25/17$.

FOUNDATION

HIGHER

Expressing Relationships

Solving Problems involving Other Sequences

The objective is to build on students' confidence in other sequences in order to use nth term rules to solve problems involving other sequences. Students should be able to explain the pattern of a sequence in words and algebraically. For example: • A sequence begins 10, 15, 21, 28, 36, ... Write down the next two terms and describe a rule to continue the sequence • The nth term rule of a sequence is $3n/(n+2)$. Show that the difference between the $(n+1)$ th term and the nth term is $6/(n^2+5n+6)$.

FOUNDATION

HIGHER

Expressing Relationships

Deduce expressions to calculate the nth term of {quadratic} sequences.

ALGEBRA

Finding the nth term of Quadratic Sequences of the form an^2

The objective is for students to find the nth term rule of a quadratic sequence of the form an^2 . Students should be able to find the second difference and write the nth term rule for a sequence of the form an^2 . For example, write down an expression for the nth term of the following sequence: 3, 12, 27, 48, 75

FOUNDATION

HIGHER

Expressing Relationships

Finding the nth term of Quadratic Sequences of the form an^2+bn

The objective is to build on students' ability of finding the nth term rule of a quadratic sequence of the form an^2 to now be able to find the nth term rule of a quadratic sequence in the form an^2+bn . Students should be able to find the second difference to find the value of a and use that to find the value of b. For example, write down an expression for the nth term of the following sequence: 5, 8, 13, 20, 29,...

FOUNDATION

HIGHER

Expressing Relationships

Finding the nth term of Quadratic Sequences of the form an^2+bn+c

The objective is to build on students' ability to write nth term rules of the form an^2+bn in order to now write nth term rules of the form an^2+bn+c . Students should be able to find the second difference and the value of a in order to find the values of b and c. For example, write down an expression for the nth term of the sequence 5, 9, 15, 23, 33, ...

FOUNDATION

HIGHER

Expressing Relationships

{Construct and interpret diagrams for grouped discrete data and continuous data, i.e. histograms with equal and unequal class intervals and cumulative frequency graphs, and know their appropriate use}

PROBABILITY AND STATISTICS

6

Cumulative Frequency
Diagrams

The objective is for students to be able to construct cumulative frequency diagrams. Students should be able to construct a cumulative frequency diagram and interpret it. For example, given grouped data of the heights of students at a school, students can construct a cumulative frequency diagram and use it to estimate the number of students who are between two heights.

HIGHER

Interpreting Data

Constructing Histograms
with Equal Class Intervals

The objective is for students to be able to construct histograms with equal class intervals. Students should be able to identify the differences between a histogram and bar chart and draw histograms with equal class intervals. For example, given grouped data on the amount of time taken to complete a times table test, students can construct a histogram.

HIGHER

Interpreting Data

Interpreting Histograms
with Equal Class Intervals

The objective builds on students' prior knowledge of constructing histograms to be able to interpret histograms with equal class intervals. Students should be able to draw conclusions about data given a histogram. For example, given a histogram showing the height of trees, students can identify which range has more or less trees.

HIGHER

Interpreting Data

Constructing Histograms
with Unequal Class
Intervals

The objective builds on students' prior knowledge of histograms in order to construct histograms with unequal class intervals. Students should understand and be able to work out frequency density in order to construct histograms with unequal class intervals. Students should also be able to group data into appropriate groups. For example, given data on the time taken to read a book grouped into unequal class intervals, students can work out the frequency densities of each class interval and construct a histogram.

HIGHER

Interpreting Data

Interpreting Histograms
with Unequal Class
Intervals

The objective builds on students' skills in constructing histograms with unequal class intervals to now be able to interpret such diagrams. Students should be able to understand and calculate the area of the bars. For example, given a histogram of time taken to complete a puzzle, students can calculate the number of people within a given range by working out the area of the bar.

HIGHER

Interpreting Data

Unit 11.3 - Grouped Frequency Data Representation

PROBABILITY AND STATISTICS

12
HOURS

{Construct and interpret diagrams for grouped discrete data and continuous data, i.e. histograms with equal and unequal class intervals and cumulative frequency graphs, and know their appropriate use}

PROBABILITY AND STATISTICS

6

Solving Problems involving Grouped Data

The objective is for students to consolidate their understanding of cumulative frequency diagrams and histograms in order to solve problems involving grouped data. Students should be able to choose an appropriate diagram, construct and interpret it in context. For example, given raw data of the time taken to complete an exercise, students can sort the data into appropriate class intervals and draw a cumulative frequency diagram in order to estimate to the percent of participants who completed the exercise in a given time.

HIGHER

Interpreting Data

Interpret, analyse and compare the distributions of data sets from univariate empirical distributions through: • appropriate graphical representation involving discrete, continuous and grouped data, {including box plots} • appropriate measures of central tendency (including modal class) and spread {including quartiles and inter-quartile range}

PROBABILITY AND STATISTICS

6

Examining & Grouping Data

The objective is to instil confidence in being able to discuss types of data and to calculate measures of central tendency and range from ungrouped data. Students should be able to decide whether data is quantitative or qualitative; discrete or continuous; and primary or secondary. They should also be able to calculate mean, median, mode and range of ungrouped data given as a list or in a frequency table. Higher students should also be able to calculate quartiles. Students should understand the difference between grouped and ungrouped data and be able to articulate advantages and disadvantages of grouping data. They should be able to group data into appropriate intervals. For example, given data on the height of plants, students can calculate mean, median, mode, and range (higher students will also calculate the quartiles) and group the data into appropriate intervals.

FOUNDATION

HIGHER

Interpreting Data

Measures of Central Tendency in Grouped Data

The objective is for students to calculate measures of central tendency from grouped data. Students should be able to identify the interval containing the median, identify the modal class and calculate an estimate for the mean given grouped data. For example, given grouped data on the weight of fish sold in a market, students can find the mean, median, modal class and range.

FOUNDATION

HIGHER

Interpreting Data

Measures of Spread

The objective is for students to examine measures of spread in data and consider outliers. Students should be able to calculate the range of a data set and look for any unusual values that do not seem to fit. Higher students should be able to calculate and discuss the meaning of inter-quartile range. For example, given race times, students can find the range (and inter-quartile range for higher students) and discuss any outliers.

FOUNDATION

HIGHER

Interpreting Data

Unit 11.3 - Grouped Frequency Data Representation

PROBABILITY AND STATISTICS

12
HOURS

Interpret, analyse and compare the distributions of data sets from univariate empirical distributions through: • appropriate graphical representation involving discrete, continuous and grouped data, {including box plots} • appropriate measures of central tendency (including modal class) and spread {including quartiles and inter-quartile range}

PROBABILITY AND STATISTICS

6

Displaying & Interpreting Data

Interpreting Data

The objective is for students to display and interpret data in a variety of formats. Students should be able to use lists, tables and diagrams to display data. Higher students should be able to use box plots and cumulative frequency diagrams to display data and be able to calculate quartiles, median and inter-quartile range from a box plot or cumulative frequency diagram. For example, given a cumulative frequency diagram of the price of cars, Higher students can find the median and inter-quartile range.

FOUNDATION

HIGHER

Displaying & Interpreting Data

Interpreting Data

The objective is for students to display and interpret data in a variety of formats. Students should be able to use lists, tables and diagrams to display data. Higher students should be able to use histograms to display data and be able to estimate the median or other information from a histogram. For example, given a histogram representing the price of dresses, Higher students can estimate the median.

FOUNDATION

HIGHER

Interpreting and Comparing Distributions

Interpreting Data

The objective is for students to compare distributions of data given in a variety of formats. Students should be able to display given data in lists, tables and diagrams in order to compare and interpret the data. For example, given marks on a test from 2 classes, students can calculate mean, median, mode and range and compare the data. Higher students can calculate quartiles and inter-quartile range and display the data on a box plot in order to compare the data.

FOUNDATION

HIGHER

use and interpret scatter graphs of bivariate data; recognise correlation and know that it does not indicate causation; draw estimated lines of best fit; make predictions; interpolate and extrapolate apparent trends whilst knowing the dangers of so doing.

PROBABILITY AND STATISTICS

2

Scatter Graphs

Interpreting Data

The objective is for students to interpret scatter graphs of bivariate data. Students should be able to read information off a given scatter graph and interpret its meaning. For example, given a scatter graph of the price of second-hand bicycles vs. their age, students can read off points indicating particular prices.

FOUNDATION

HIGHER

Scatter Graphs

Interpreting Data

The objective is for students to draw scatter graphs of bivariate data. Students should be able to draw a scatter graph given data. For example, given data on mock vs. final exam grades, students can plot a scatter graph and begin to comment on a general trend in the data.

FOUNDATION

HIGHER

Recognise correlation and know that it does not indicate causation

PROBABILITY AND STATISTICS

1

Correlation

Interpreting Data

The objective is for students to recognise correlation. Students should be able to recognise and name strong, weak, moderate, positive, negative and no correlation. Students should understand that correlation does not equal causation. For example, given a scatter diagram of Maths and English test scores, students can say whether the correlation is strong, weak or moderate; positive, negative or no correlation; and interpret any correlation in context.

FOUNDATION

HIGHER

Draw estimated lines of best fit; make predictions; interpolate and extrapolate apparent trends whilst knowing

PROBABILITY AND STATISTICS

1

Lines of Best Fit

Interpreting Data

The objective is for students to draw and interpret lines of best fit. Students should be able to draw a line of best fit by eye for data with a strong correlation and understand the effect of outliers on drawing a line of best fit. They should also be able to interpolate and extrapolate trends using the line of best fit. For example, given a scatter diagram of height and weight, students can draw a line of best fit and estimate a weight given a particular height while identifying the possible dangers of doing so.

FOUNDATION

HIGHER

Infer properties of populations or distributions from a sample, whilst knowing the limitations of sampling

PROBABILITY AND STATISTICS

1

Sampling

Interpreting Data

The objective is for students to infer properties of populations from a sample. Students should be able to find patterns in data, look for unusual values and infer properties of populations from the data. They should understand that samples may or may not be representative of a population and understand the constraints of sampling. For example: • Given the number of students who purchase school dinners in one class, students can estimate the number of students in the school who purchase school dinners • Given data of the amount of wrapping paper purchased in December, students understand that is not indicative of a trend throughout the year.

FOUNDATION

HIGHER

Interpret and construct tables and line graphs for time series data

PROBABILITY AND STATISTICS

1

Time Series Data

Interpreting Data

The objective is for students to construct and interpret tables and line graphs for time series data. Students should understand the definition of time series; be able to plot and interpret time-series graphs; and use a time-series graph to make predictions. For example, given data on sales of ice cream throughout the year, students can construct a table and a line graph for the data. Students can then comment on general trends and make general predictions of sales for the following year.

FOUNDATION

HIGHER

Apply statistics to describe a population

PROBABILITY AND STATISTICS

1

Describing Populations

Interpreting Data

The objective is for students to describe a population using statistics. Students should be able to use measures of central tendency and dispersion in addition to statistical diagrams in order to describe a population. For example, given data on percentage of male and female shoppers, their mean ages and the range, students can interpret the data and describe the population of shoppers.

FOUNDATION

HIGHER

Interpret and use fractional {and negative} scale factors for enlargements

GEOMETRY AND MEASURES

4

Enlargement

Measures

The objective is to instil confidence in enlargement on a grid using a centre of enlargement. Students should be able to enlarge a shape given a centre of enlargement and a scale factor that is a positive integer. Students should be able to make the link between the scale factor and the ratio of the enlargement. Students should recognise that enlargements are similar shapes. For example, given a triangle on a coordinate grid, the centre of enlargement and a scale factor of 2, students can draw the image on the grid and identify that the ratio of the object to its image is 1:2.

FOUNDATION

HIGHER

Fractional Scale Factors

Measures

The objective is for students to understand fractional scale factors. Students should be able to construct an enlargement on a grid given a centre of enlargement and a positive fractional scale factor. Students should make the link between a fractional scale factor and the ratio of enlargement. For example, given a rectangle on a coordinate grid, the centre of enlargement and a scale factor of $1/2$, students can draw the image on the grid and articulate that the ratio of the object to its image is 2:1.

FOUNDATION

HIGHER

Negative Scale Factors

Measures

The objective is to introduce students to negative scale factors. Students should be able to construct an enlargement on a grid given a negative scale factor. For example, given a trapezium on a coordinate grid, a centre of enlargement and a scale factor of -3, students can draw the image on the grid.

HIGHER

Describing an
Enlargement

Measures

The objective is for students to describe a given enlargement. Students should be able to find the scale factor and centre of enlargement. For example, given a triangle and its enlarged image on a coordinate grid, students can calculate the scale factor and find the centre of enlargement.

FOUNDATION

HIGHER

{Describe the changes and invariance achieved by combinations of rotations, reflections and translations}

GEOMETRY AND MEASURES

3

Transformations

Position and Direction

The objective is to recap and instil confidence in rotation, reflection and translation of shapes. Students should be able to reflect a shape given a mirror line; rotate a shape given the angle and centre of rotation; and translate a shape. They should also understand and use the term “invariance” for points, lines and shapes. For example, given a parallelogram on a coordinate grid, a centre of rotation and an angle of 90 degrees, students can draw the image on the coordinate grid and identify any points that did not change.

HIGHER

Combining Transformations

Position and Direction

The objective is for students to combine transformations. Students should be able to combine rotations, reflections and translations of shapes. For example, given a triangle on a grid, students can reflect then rotate the triangle and draw the image.

HIGHER

Describing Transformations

Position and Direction

The objective is for students to describe a combination of transformations as a single transformation. Students should be able to recognise when a shape has been reflected, rotated, or translated. Students should be able to describe the transformation or combination of transformations that would map an image back to the object. For example, given L-shaped polygons on a grid, students can identify the transformations that have mapped the object to the image.

HIGHER

Describe translations as 2D vectors

GEOMETRY AND MEASURES

2

Translating Shapes using Vectors

Position and Direction

The objective is for students to understand that a vector can be used to specify the distance and direction of a translation. Students should be able to translate a shape given a vector. For example, given a shape on a coordinate grid and vector indicating a shift of +4 in the x-direction and -2 in the y-direction, students can draw the image of the shape.

FOUNDATION

HIGHER

Describing Translations using Vectors

Position and Direction

The objective is for students to describe translations using vectors. Students should be able to recognise a translation, identify the distance and direction and express the translation as a vector. For example, given an object and its translated image, students can articulate the translation that has occurred as a vector.

FOUNDATION

HIGHER

Apply addition and subtraction of vectors, multiplication of vectors by a scalar, and diagrammatic and column representations of vectors; {use vectors to construct geometric arguments and proofs}.

GEOMETRY AND MEASURES

3

Representing Vectors

Position and Direction

The objective is for students to understand and use vector notation. Students should make the link between vectors used to describe translations and the idea that a vector describes movement in a fixed distance and direction. Students should be able to represent vectors diagrammatically and as columns. For example, given a shape drawn on isometric paper, students can identify the vectors between various points and express them using vector notation.

FOUNDATION

HIGHER

Addition and Subtraction of Vectors

Position and Direction

The objective is for students to calculate and represent graphically the sum or difference of two vectors and to calculate the resultant vector. Students should understand the commutative and associative properties of vector addition. Students should be able to draw diagrams representing the sum or difference of given sums of vectors. They should also be able to represent a given diagram as the sum or difference of vectors. For example, given a rectangle, students can work out the vector that represents the diagonal.

FOUNDATION

HIGHER

Multiplication of Vectors by a Scale Factor

Position and Direction

The objective is for students to calculate a scale multiple of a vector and represent it graphically. Students should understand that a scalar has magnitude but no direction and that scalar multiples of vectors are parallel. For example, given a diagram of points on a grid, students can express distances between the points as scalar multiples of a given vector.

FOUNDATION

HIGHER

Combining Vectors

Position and Direction

The objective is to consolidate students' understanding of sums and scalar multiples of vectors. Students should be able to combine sums and scalar multiples of vectors. For example, given two vectors a and b , students can draw $3a+4b$ and $3(a+b)$.

FOUNDATION

HIGHER

Vectors in Geometry

Position and Direction

The objective is for students to identify vectors in geometry. Students should be able to identify vectors and combinations of vectors in shapes. Higher students should be able to solve simple geometrical problems using vector methods. For example, •Given a quadrilateral ABCD and properties of its vectors, students can name the type of quadrilateral •(Higher) Given a square ABCD with vectors between A and the midpoint e between AB and f between DA, students can express the diagonal AC in terms of the vectors.

FOUNDATION

HIGHER

Proof using Vectors

Position and Direction

The objective builds on students' understanding of vectors in geometry to be able to construct geometrical arguments and proofs using vectors. Students should be able to identify vectors and combinations of vectors in shapes in order to prove geometrical properties. Students should recognise and be able to show when lines are parallel and when points are collinear using vectors. For example, given a rectangle ABCD, students can prove that the line joining the midpoints of AB and BC is parallel to the diagonal AC.

HIGHER

Construct and interpret plans and elevations of 3D shapes

GEOMETRY AND MEASURES

1

Plans and Elevations of 3D Shapes

The objective is for students to construct and interpret plans and elevations of 3D shapes. Students should be able to understand and draw front and side elevations and plans of shapes made from simple solids. For example, students can draw the plan, front and side elevations of a shape composed of small cubes.

Properties of Shapes

FOUNDATION

HIGHER

Understand and use coordinates in 3 dimensions

GEOMETRY AND MEASURES

1

Coordinates in 3D

The objective is for students to understand and use coordinates in 3D. Students should be able to plot a point given 3D coordinates and recognise the coordinates of a given point. For example, given the point (2,3,5), students can plot it on a 3D grid.

Position and Direction

NON-STATUTORY

Solve linear inequalities in one {or two} variable{s}, {and quadratic inequalities in one variable}; represent the solution set on a number line, {using set notation and on a graph}

ALGEBRA

6

Inequalities on a Number Line

Expressing Relationships

The objective is for students to be able to represent inequalities on a number line. Students should know meaning of inequality symbols and be able to represent them using conventions of open and closed circles on a number line. Higher students should be able to express inequalities using set notation. For example: • Students can show the inequality $3 < x < 7$ on a number line with an open circles and indicate the values between 3 and 7. •(Higher) Students can express the integer values of the inequality $3 < x < 7$ as {4, 5, 6}

FOUNDATION

HIGHER

Solving Linear Inequalities

Expressing Relationships

The objective is for students to solve linear inequalities in one variable. Students should be able to solve simple inequalities and represent solutions on a number line. Higher students should be able to solve more advanced linear inequalities and represent solutions on a number line and using set notation. For example: •Students can solve the inequality $2x+4 < 10$ and represent the solutions on a number line. •(Higher) Students can solve the inequality $2(x+4) < 3(x-8)$ and represent the solutions on a number line

FOUNDATION

HIGHER

Representing Inequalities Graphically

Using Equations, Functions and Graphs

The objective is for students to represent inequalities by shading regions on a graph. Students should be able to recognise the boundary line and shade the side that does not satisfy the inequality. Students should recognise and use the convention of dashed and solid lines for inequalities. For example, given the inequality $y < 2x+4$, students can draw a dashed line at $y=2x+4$ and shade the region above the line.

HIGHER

Solving Inequalities in Two Variables

Using Equations, Functions and Graphs

The objective is for students to solve inequalities in two variables. Students should be able to represent the solution set to inequalities in 2 variables on a graph. For example, students should be able to represent the set of solutions that satisfy the inequalities $y < x+8$, $y > 4$ and $x < 2$ on a graph.

HIGHER

Solving Quadratic Inequalities

Using Equations, Functions and Graphs

The objective is for students to solve quadratic inequalities. Students should be able to represent the solution to a quadratic inequality on a graph and a number line. For example, students should be able to represent the set of solutions to $x^2+7x+12 > 0$ on a graph and a number line.

HIGHER

Solving Problems involving Inequalities

Using Equations and Functions

The objective is for students to be able to solve problems involving inequalities in context. Students should be able to write inequalities, solve them, and use appropriate methods to display their solutions. For example, given a rectangle with dimensions in x and restrictions for its perimeter, students can write an inequality, solve for possible values of x and represent them on a number line.

FOUNDATION

HIGHER

Know the difference between an equation and an identity; argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments {and proofs}

ALGEBRA

2

Equations and Identities

The objective is for students to understand the difference between an equation and an identity and to use identities including equating coefficients. Students should be able to show algebraically that expressions are equivalent. For example, students can work out the values of a and b in the identity $4(ax-1)+5(2x+b)=22x+6$.

Using Equations and Functions

FOUNDATION

HIGHER

Mathematical Arguments

The objective is for students to make mathematical arguments. Students should be able to use algebra and algebraic expressions to construct arguments. Higher students should be able to construct rigorous proofs. For example, given that x is an odd number, students can explain why $(x-1)(x+1)$ will be even. •(Higher) Students can prove using algebraic expressions that the sum of two consecutive positive integers is always odd.

Expressing Relationships

FOUNDATION

HIGHER

Use algebraic and geometric understanding to construct and interpret mathematical proof

ALGEBRA

2

GEOMETRY AND MEASURES

Geometric Proofs

The objective is for students to understand and construct geometric proofs. Students should be able to apply their knowledge of angle facts and similarity in order to prove geometric properties. Students should be able to present their proofs with clear, logical steps and reasons. For example, students can understand and construct the proof that the angles in a triangle sum to 180 degrees.

Geometry

FOUNDATION

HIGHER

Algebraic Proofs

The objective is for students to understand and construct algebraic proofs. Students should be able to use algebraic and numerical properties to prove statements. Higher students should be able to construct rigorous proofs. For example, given that x is an odd number, students can prove that the square of any odd number is always one more than a multiple of 8.

Expressing Relationships

FOUNDATION

HIGHER

{Change recurring decimals into their corresponding fractions and vice versa}

NUMBER

2

Converting Recurring
Decimals to Fractions

The objective is for students to convert recurring decimals to fractions. Students should be able to use formal algebraic proof to convert recurring decimals into fractions. For example, students should be able to convert 0.39 (39 recurring) into a fraction.

HIGHER

Fractions (Including Decimals)

Converting Fractions to
Recurring Decimals

The objective is for students to convert fraction into recurring decimals. Students should be able to identify a fraction that will result in a recurring decimal and convert it. For example, students can convert $\frac{3}{11}$ into a recurring decimal.

HIGHER

Fractions (Including Decimals)

Calculate arc lengths, angles and areas of sectors of circles

GEOMETRY AND MEASURES

3

Arc Length

Geometry

The objective is for students to calculate the length of arcs. Students should be able to calculate an arc length given the angle subtended at the centre. They should also be able to calculate the angle subtended at the centre given the length of an arc and the circumference. For example, given an angle of 130 degrees and a radius of 5 cm, students can work out the length of the arc.

FOUNDATION

HIGHER

Sector Area

Geometry

The objective is for students to calculate sector areas. Students should be able to calculate the area of sectors of circles. They should also be able to calculate the angle subtended at the centre given the sector area and area of the circle. For example, given the sector area of 130 cm² and the radius of 7 cm, students can calculate the angle subtended at the centre.

FOUNDATION

HIGHER

Solving Problems involving Arc Length and Sector Area

Geometry

The objective is for students to solve problems involving arc length and sector area. Students should be able to apply their knowledge of arc length and sector area in context. For example, given an archway constructed from two circles, students can calculate the area of the arch given the angle subtended at the centre and the radii of the circles.

FOUNDATION

HIGHER

Calculate surface areas and volumes of spheres, pyramids, cones and composite solids

GEOMETRY AND MEASURES

3

Volume

Properties of Shapes

The objective is for students to calculate volume of solids. Students should be able to calculate the volume of spheres, pyramids, cones and composite solids. For example, students can calculate the volume of solid made up of a sphere and a cylinder.

FOUNDATION

HIGHER

Surface Area

Properties of Shapes

The objective is for students to calculate surface area of solids. Students should be able to calculate the surface area of spheres, pyramids, cones and composite solids. For example, students can calculate the surface area of solid made up of a frustum and a cylinder.

FOUNDATION

HIGHER

Frustums

Properties of Shapes

The objective is for students to calculate volume and surface area of frustums. Students should be able to calculate the volume and surface area of cones and apply this knowledge to finding the volume and surface area of frustums. For example, given a frustum with base radius 10 cm, top radius 5 cm and a height of 3 cm, students can calculate the volume and surface area.

FOUNDATION

HIGHER

Mathematics. Solved.