Complete Mathematics

Problem Solving Booklet

La Salle EDUCATION

Complete Mathematics

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Contents

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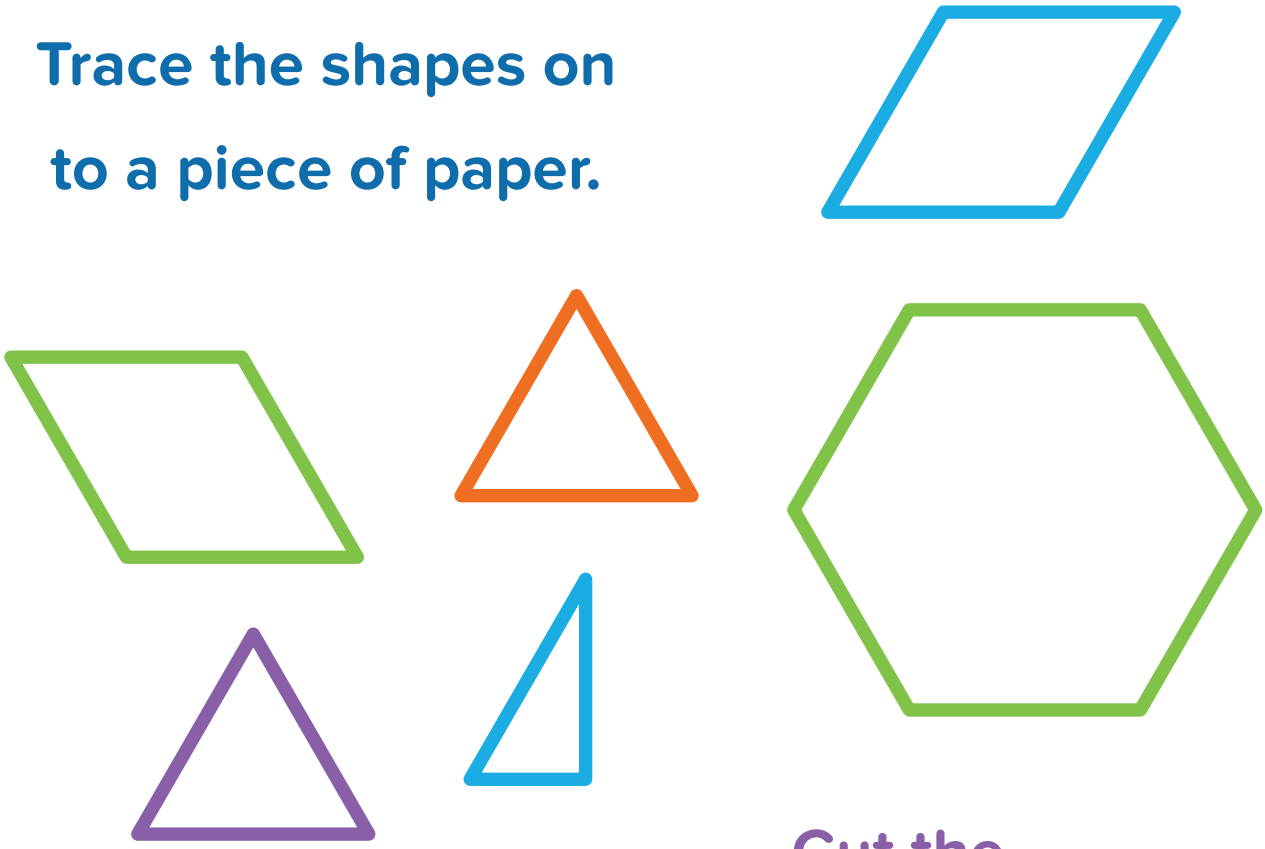
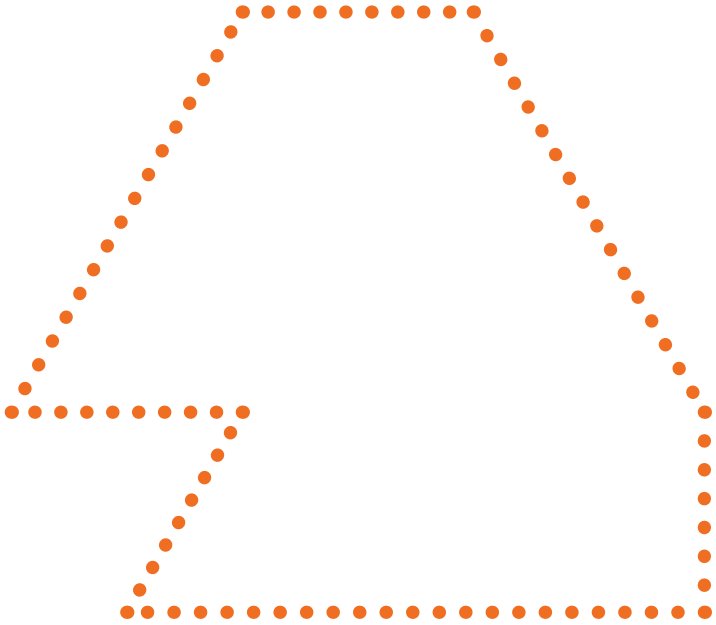
Tangrams

Trace the shapes on to a piece of paper.

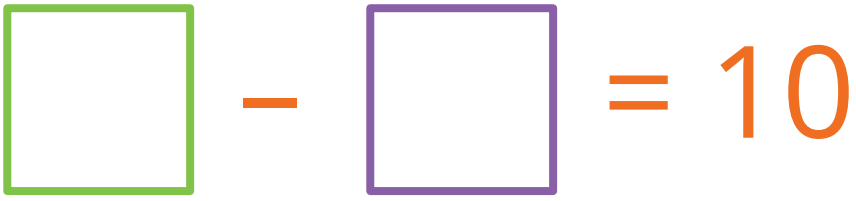


Cut the shapes out.



Can you get them to fit perfectly into this shape?

10 - No More, No Less

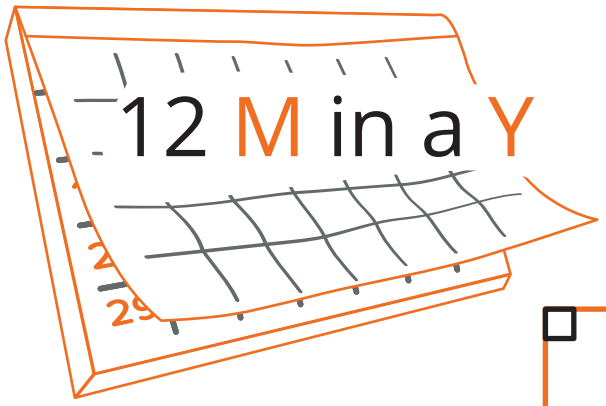


How many ways can you complete this puzzle?

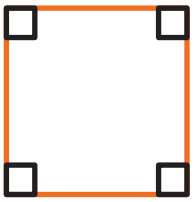
What do you notice about your answers?

One Minute Challenge 1

If a **P** has 5 **S** stands for
"a **Pentagon** has 5 **Sides**"
What do the following mean?



a P is
used to measure A



a s has 4 RA

60 M in an H

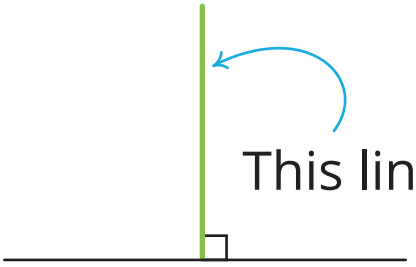
V, X & C
are examples of RN

16,24,32,40 are M of E

1/2

is the N

is the D



This line is P

Now make up **two** of your own.

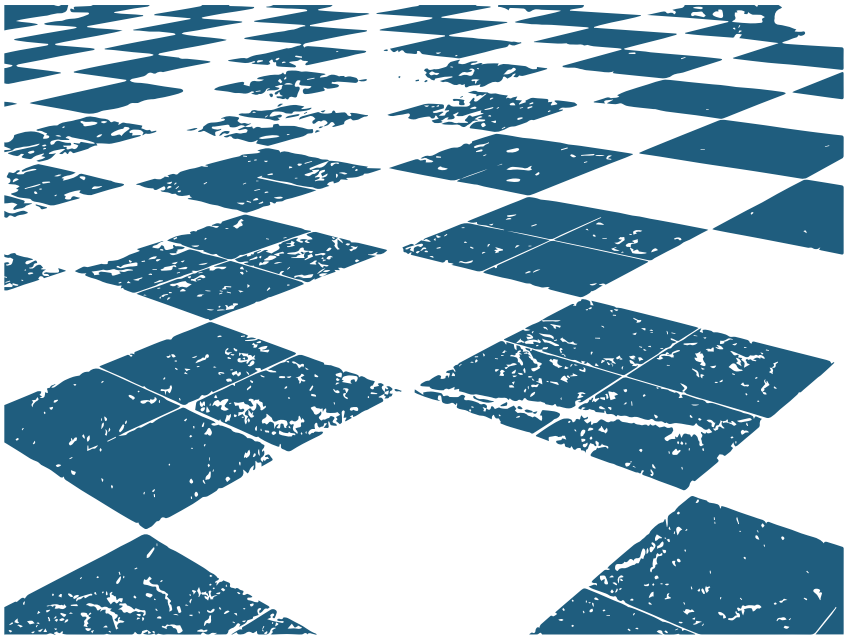
Patio Problem

I need to replace the slabs on my patio.

I need 78 new slabs to cover my patio.

The slabs come in packs of 12.

The slabs cost £25.95 per pack.

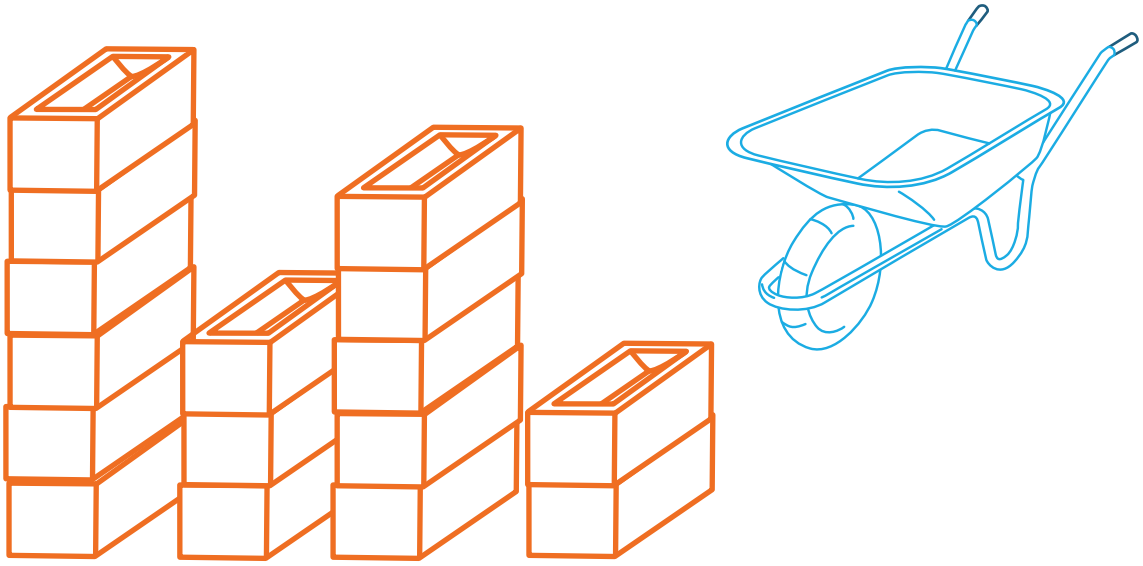


1. How many packs will I need?

2. How much will the slabs cost me?

Barry's Bricks

Barry the builder likes to keep his bricks neat and tidy, but someone has messed up his bricks!



He can move one or more bricks at a time.

In only two moves, how can Barry make all the piles the same height?

Number 7

A number with a
7 in the hundreds
position
is greater
than a number
6 in the hundreds
position.

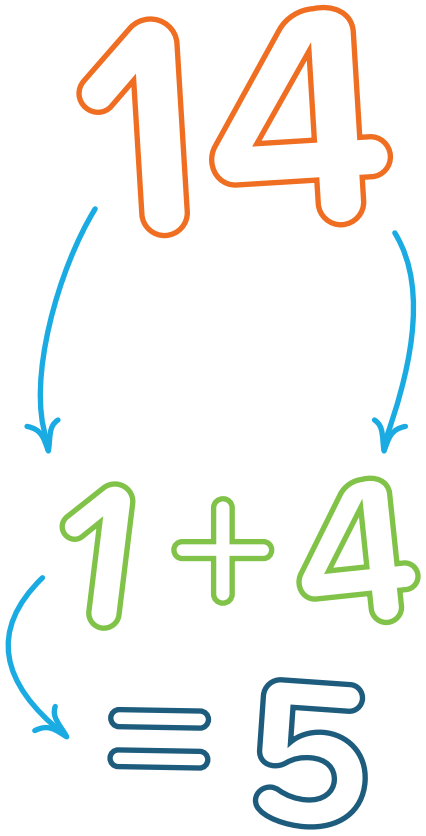
Is this statement
always/sometimes/never
true?

Give examples
to support your
response.

Total of 5

I have written a list of different 2 digit whole numbers.

The digits of each number add up to 5.

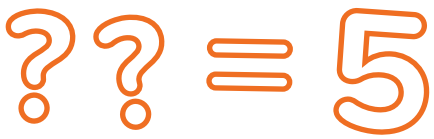


None of the digits is zero

One of my numbers is 14

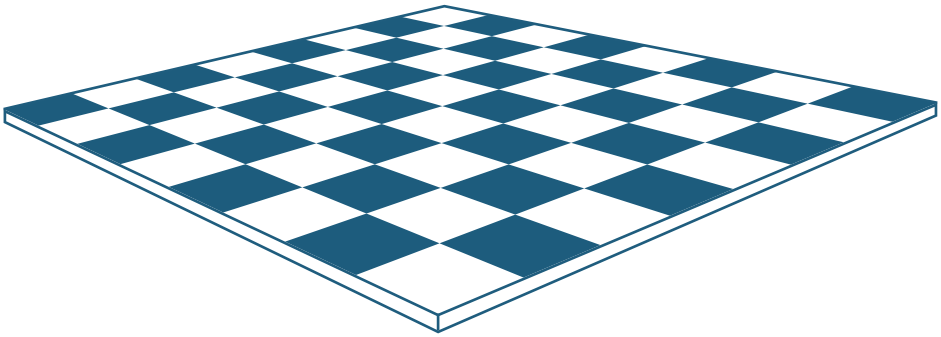
How many different numbers could I have written down?

Can you write them in order smallest to largest?



Chessboard Squares

On a standard 8X8 chessboard how many squares are there?

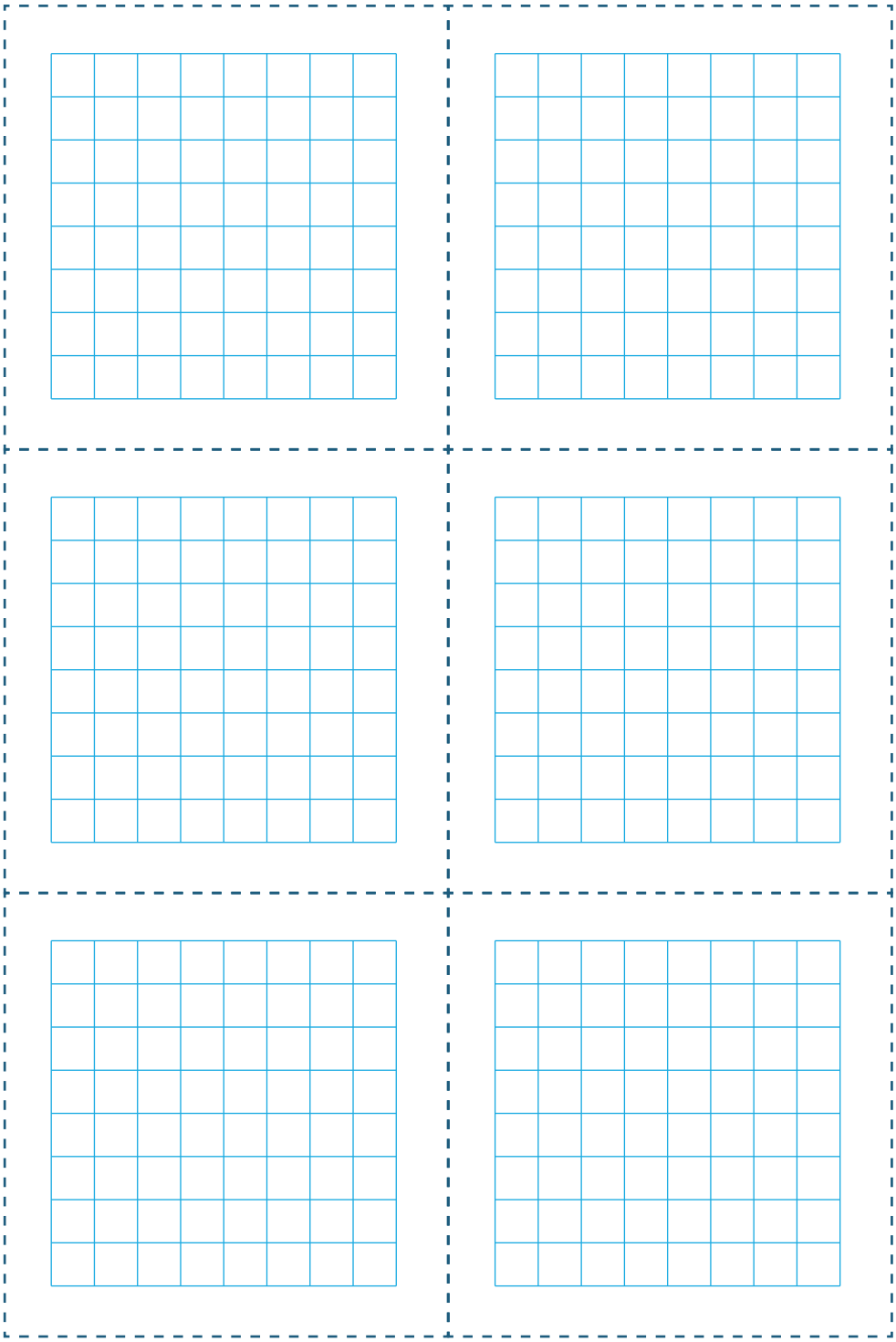


How many different sized squares are there?

Can you see any patterns?

Could you predict how many square there would be if the board was 10X10?

Chessboard Squares



Consecutive Sums

Investigate the properties of numbers that can be represented as sums of consecutive whole numbers.

For example:

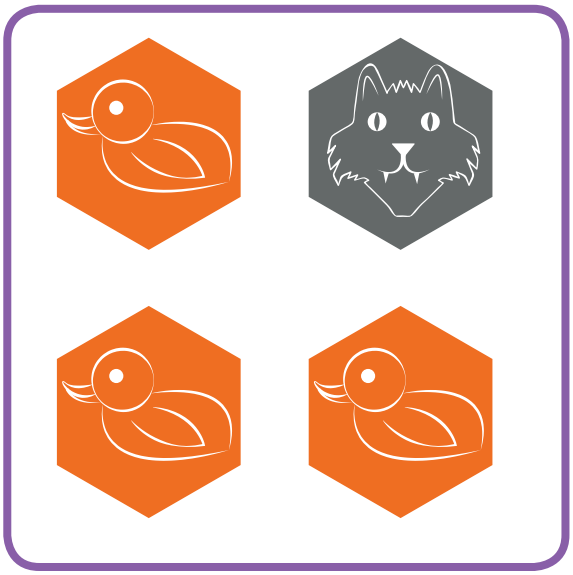
$9 = 5 + 4$

$= 4 + 3 + 2$

Ducks, Wolves and Elephants

Two boxes of toys contain the following:

Box 1



Box 2

One toy is taken from each box and the type of animals are recorded.

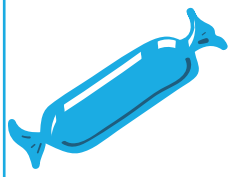
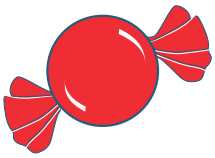
For example:

I pull a Duck out of the first box and a Wolf out of the second box. I therefore have...

- List all the other combinations of animals that I could end up with.
- How many are there?
- How can we be sure we have considered all options?

How Many Sweets?

A pie chart is being prepared, showing the numbers of various sweets in a tin. The table shows the angles for the four main types. The "other" section represents 12 sweets.

Sweet Type	 Jellies	 Minties	 Chockblocks	 Chewsies	Other
Pie Chart Angle	**90°**	**135°**	**60°**	**45°**	**?**

How many sweets are there altogether in the tin?

Population Pizza!

Look at this leaflet for Population Pizza - is their claim correct?

Choose From 26 Toppings!

Create your own pizza

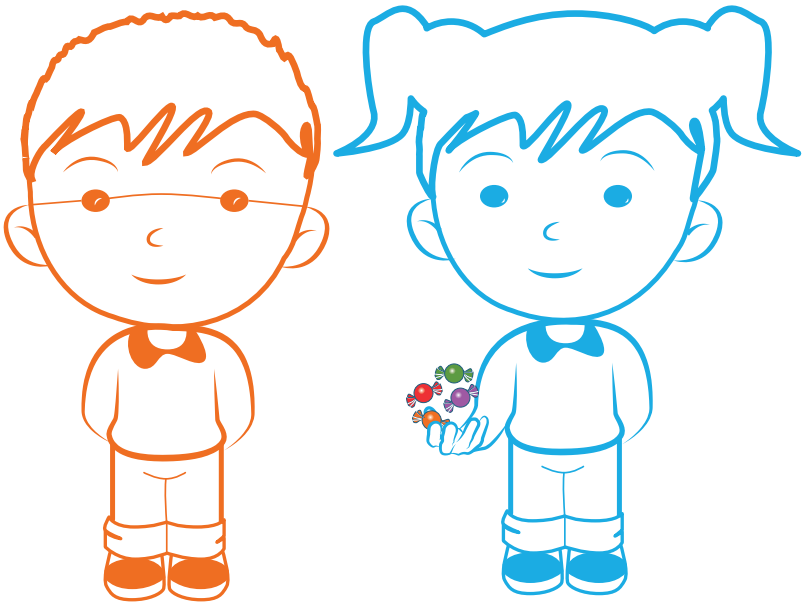
more combinations than people in the world

- BBQ sauce
- Sliced Peppers
- Pulled Pork
- Meatballs
- Chorizo
- Cumberland Sausage
- Sliced Beef
- Pepperoni
- Pineapple
- Sweetcorn
- Red Onions
- Tomatoes
- Chili Peppers
- Salmon
- Egg
- Mushrooms
- Extra cheese
- Ham
- Chicken Breast Strips
- Spicy chicken
- Crispy Bacon Bits
- Dates
- Olives
- Mixed Herbs
- Sundried Tomatoes

Samantha's Sweets

Samantha has a bag of sweets.

She gives half of them to Steven.



Steven eats two and has four left.

How many sweets did Samantha have to begin with?

The 100 Quiz

Easy – One point each

100 pence =

100cm =

100 years =

The total amount =

Medium – Two points each

What letter represents 100 in Roman Numerals?

√100= ?

True or false; the sum of the first nine prime numbers is 100?

Hard – Three points each

Find all the factors of 100

Show the prime factors of 100

Show the prime factors of 100 in exponential form

Show the number 100 in binary form

Totalling Odd Numbers

Add up all the odd numbers less than 10

What do you notice?

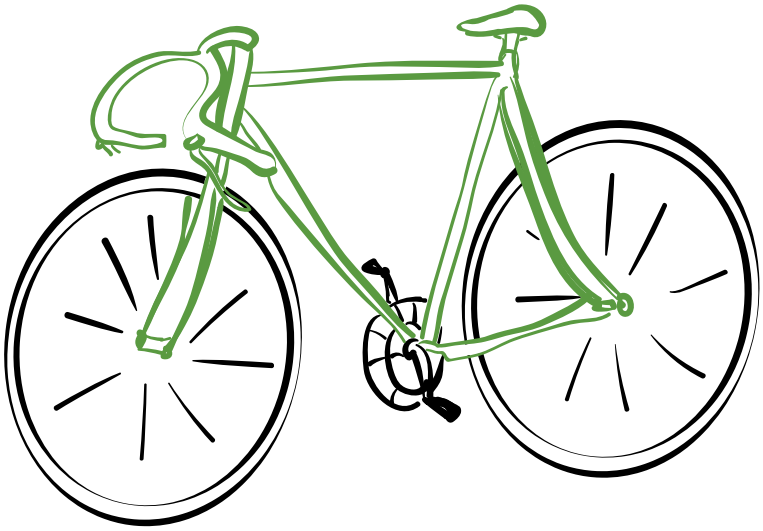
Wheelzr'us

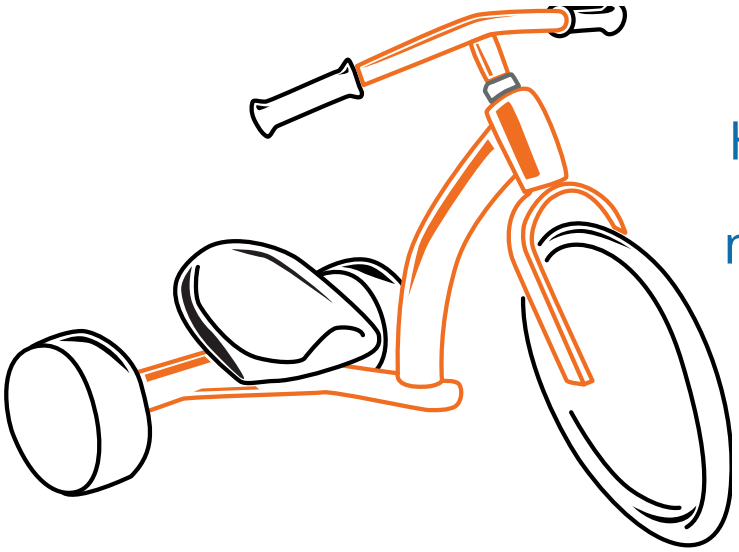
Wheelzr'us has a number of bicycles and tricycles for sale.

Lucy counted a total of

60

wheels.





How many bikes and how many trikes were for sale?

Is there more than one solution?

7's in 999

When the numbers from

1 to 999

are written out in full,

how many 7 's are used?

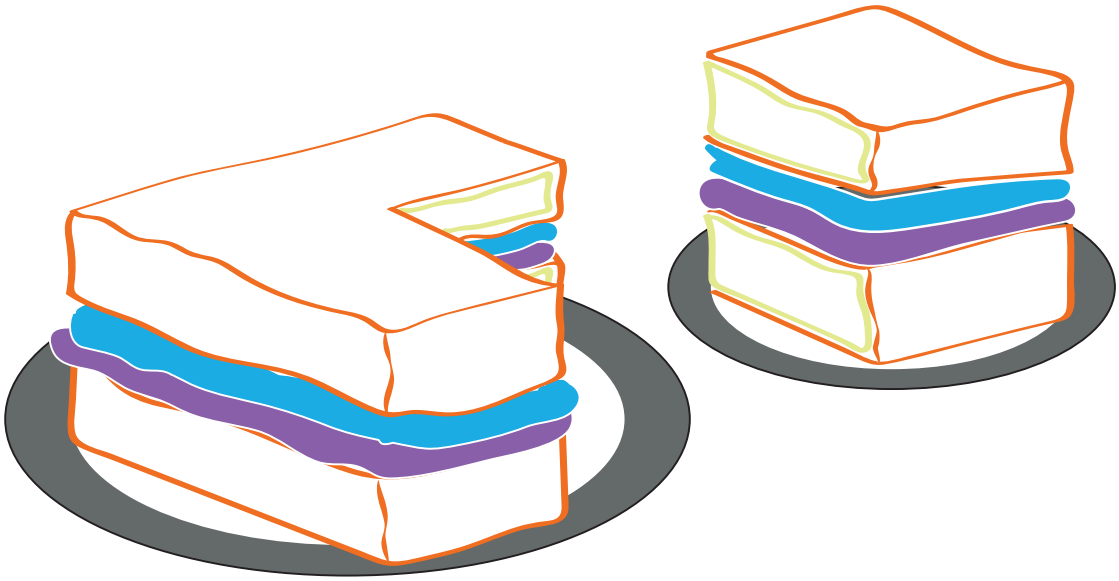
Amy's Cake

Amy bakes a square cake.

She takes $\frac{1}{4}$ of it to a friend who is unwell.

She then wants to share equally the remainder between 4 friends – but they all want the same shape piece of cake.

How does she divide up the cake?

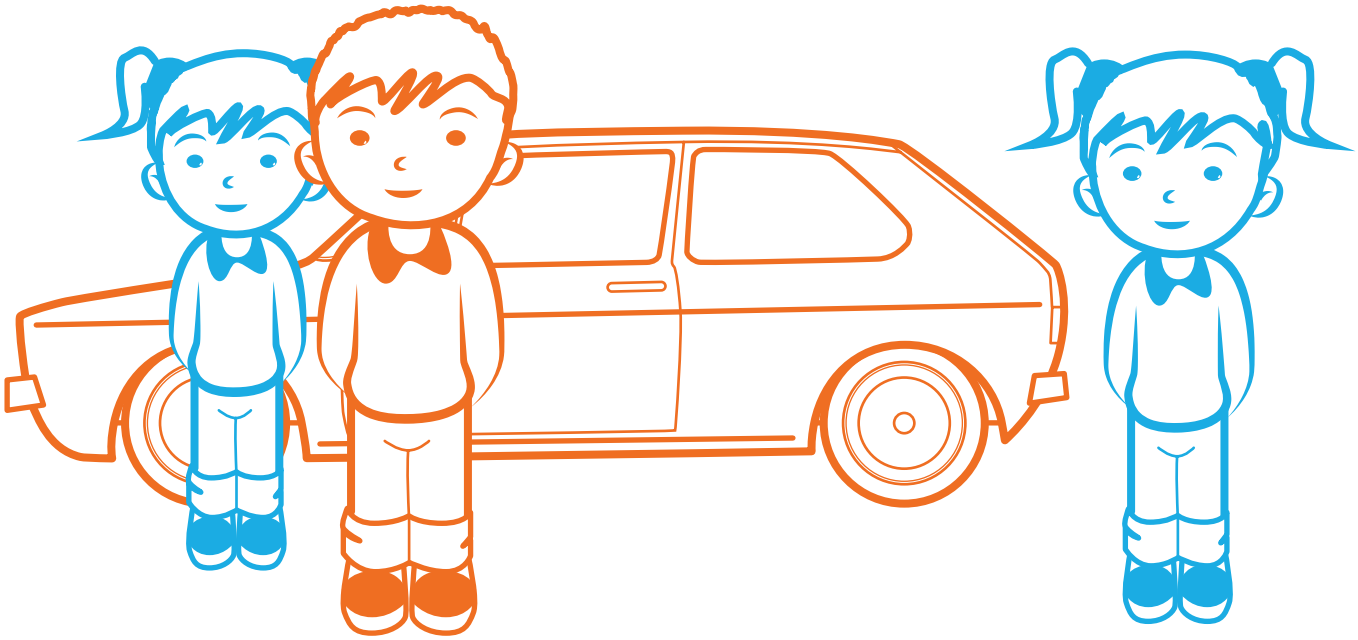


Car Percentage

Rachel bought a car for £3200 from Rose for $\frac{2}{3}$ of the amount Rose paid.

Rose bought the car from Jake for 0.75 of what he paid.

Jake bought the car from Karen for 64% of what she paid when she bought it new from the dealership.



a. How much did Karen pay?

b. What percent of the original value did Rachel pay?

Cutting Corners

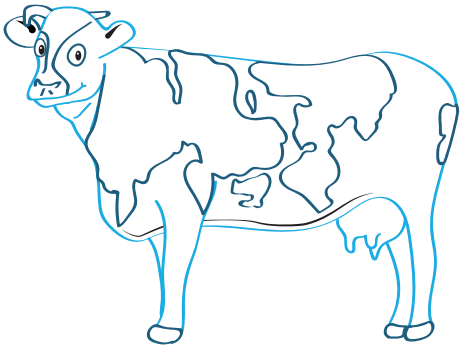
Imagine a Cube

Make a diagonal cut to slice off each vertex.

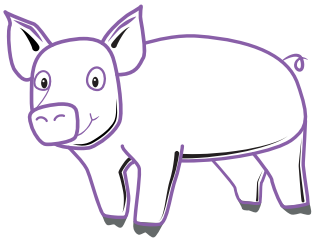
How many Verticies does the new shape have?

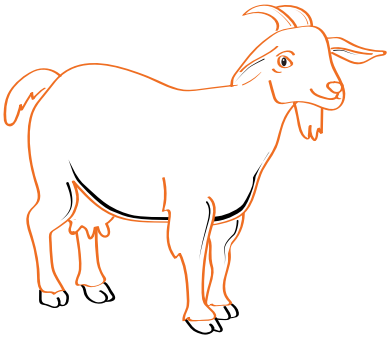
Farmer Bob Goes To Market

Farmer Bob goes to a market where the items are exchanged as shown.



= 5 ×





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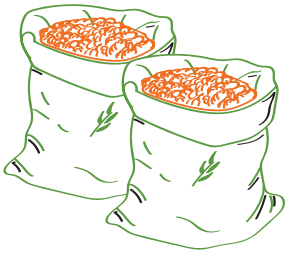
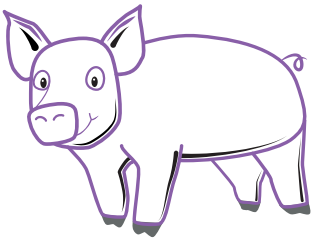


= 3 ×



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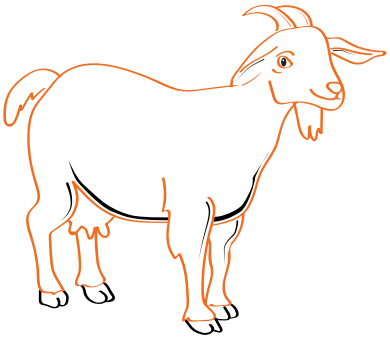


Farmer Bob wants 1 goat, 1 piglet and 1 cow.

What is the minimum number of sacks of grain he must bring to the market?

Four 4s

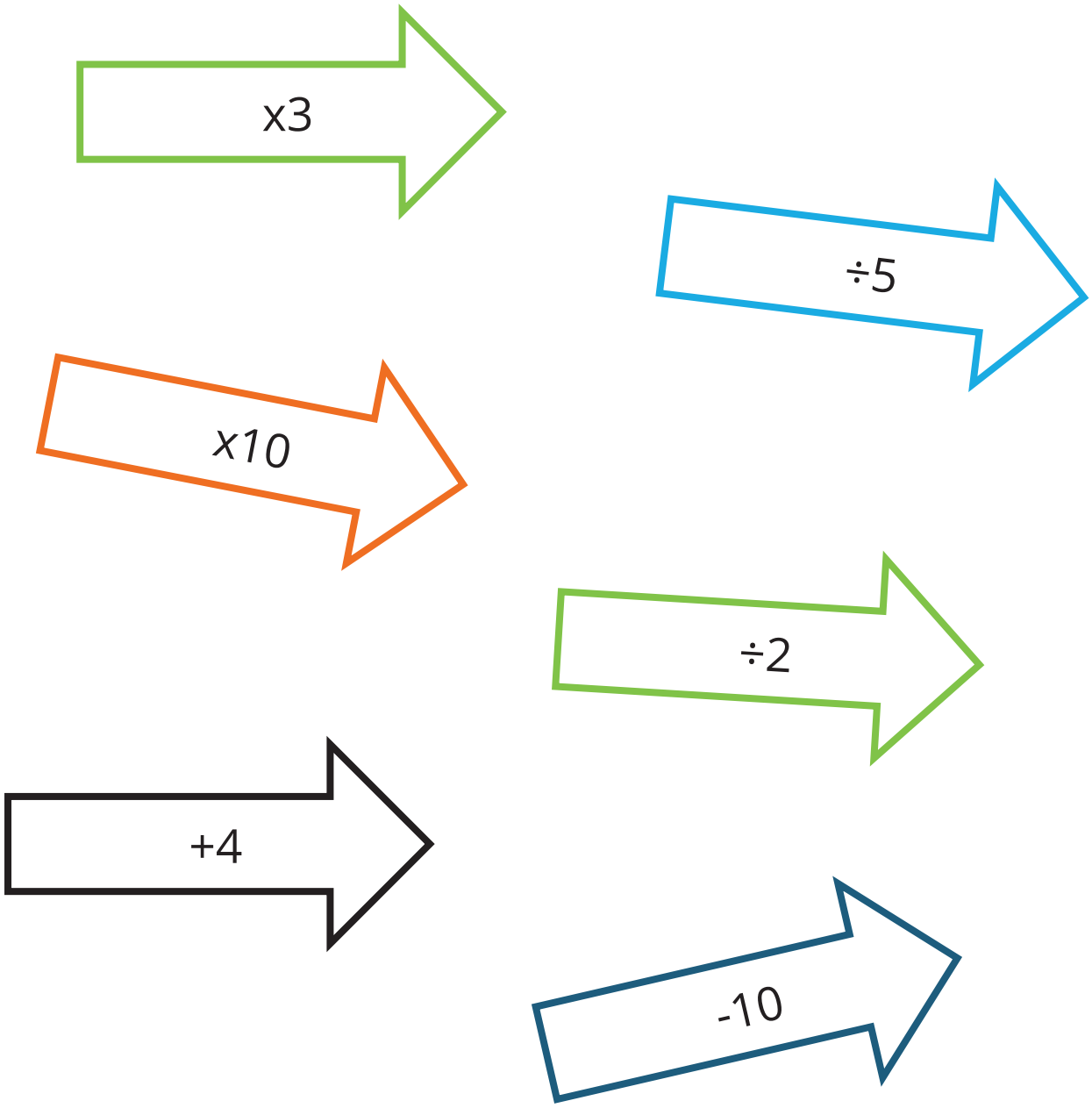
Using exactly four 4s and no other digits, along with as many of the following symbols as you need...

$+$ $\times$ $-$ $($ $)$ $\div$

...make **all** the numbers between **1** and **20**

How Low Can You Go?

Arrange these function machines so that when the input is 100 the outcome is as low as possible?



New Kettle

Bob says the new kettle holds 2000 ml of water when full, but Belle does not agree.

At the moment it is 80% full. After 20% of the water in it has been poured out, there are 1152 ml of water left.

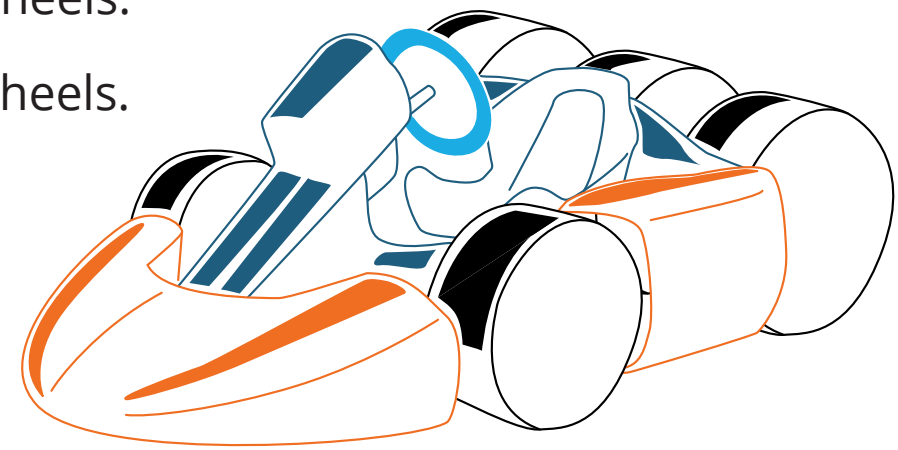
Is Bob correct?

The Toy Shop

The toy shop sells tricycles and go-carts.

The tricycles have 3 wheels.

The go-carts have 5 wheels.

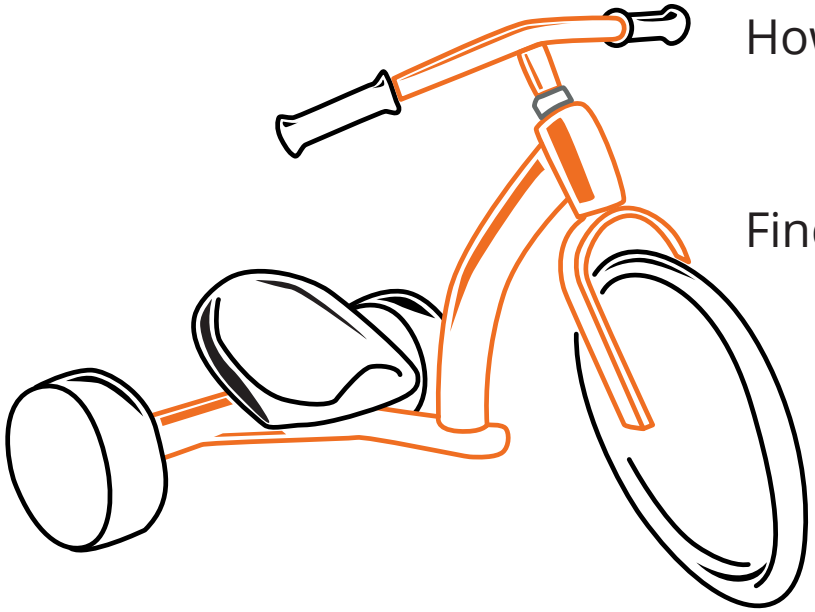


Sarah counted the wheels.

She counted 51 altogether

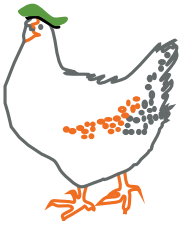
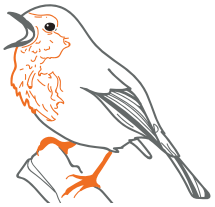
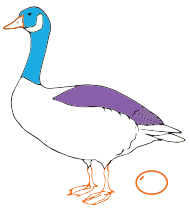
How many tricycles are there?

How many go-carts are there?



Find three ways to do it.

12 Days of Christmas

Price of one item from the gift in 2014		Percentage increase on last year
A partridge in a pear tree  $207.68 | 3.8%	Two turtle doves  $62.50 | 0%	Three French hens  $60.50 | 10%
Four calling birds  $149.99 | 0%	Five gold rings  $150.00 | 0%	Six geese a-laying  $60.00 | 71.4%
Seven swans a-swimming  $1,000.00 | 0%	Eight maids a-milking  $7.25 | 0%	Nine ladies dancing  $839.20 | 0%
Ten lords a-leaping  $534.82 | 2%	Eleven pipers piping  $239.56 | 0%	Twelve drummers drumming  $237.90 | 0%

1. How many gifts in total would you receive in the entire song?

2. The individual prices are shown in US dollars, what is the full cost in UK pounds?

3. What would the total cost in 2013 have been in UK pounds?

4. What is the total percentage increase from 2013 to 2014?

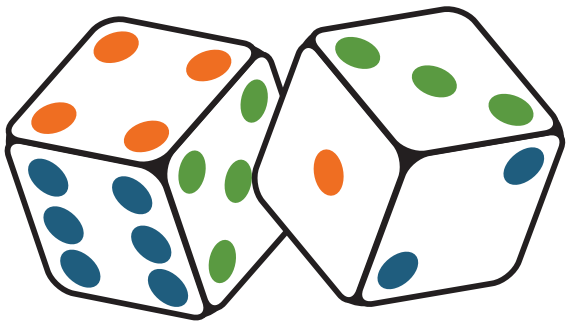
12 Days of Christmas

Day	Gift	Price of one gift item	Number of days given	Number of gifts required	Cost
1	A partridge in a pear tree	$207.68			
2	Two turtle doves	$62.50			
3	Three French hens	$60.50			
4	Four calling birds	$149.99			
5	Five gold rings	$150.00			
6	Six geese a-laying	$60.00			
7	Seven swans a-swimming	$1,000.00			
8	Eight maids a-milking	$7.25			
9	Nine ladies dancing	$839.20			
10	Ten lords a-leaping	$534.82			
11	Eleven pipers piping	$239.56			
12	Twelve drummers drumming	$237.90			
				Total Cost:	

150 Dice

150 dice show a total score of 441

What is the total of the 150 numbers which are hidden underneath each of these dice?



Aircraft Luggage

When travelling by aircraft there is a maximum allowable weight for luggage. Passengers are charged £10 for every kilogram overweight.

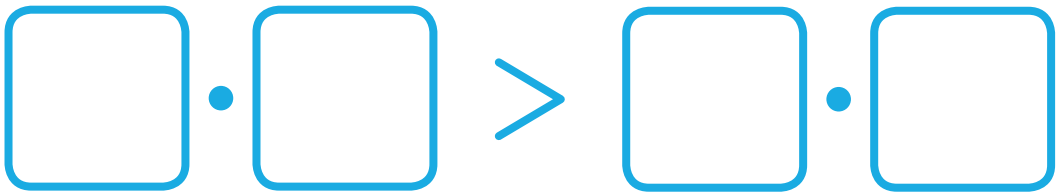
If a passenger carrying 40kg of luggage is charged £50, how much would a passenger carrying 80kg be charged?

Decimal Disposition

Using the digits...

2, 3, 7, and 8

...how many ways can you make the following statement true?



Just Make One

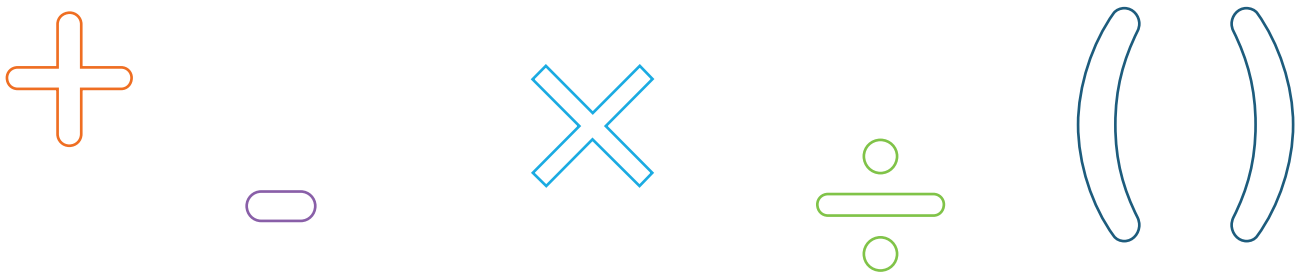
Using

6, 6, 7, 5

and

- + × ÷ ( )

Can you make the answer 1?



Mathematical Mind Reader

I'm thinking of an odd number

It is a factor of 36

It is larger than 5

What number am I thinking of ?

Meet The Joneses

Mr & Mrs Jones have three children: Tom the youngest, Georgia in the middle and Fiona the eldest.

Mr Jones is four years older than Mrs Jones. Their ages are both even numbers greater than 40.

Fiona is half the age of her mother.

Tom is a quarter of the age of his father and his age is a prime number below 20

Georgia is six years older than Tom and her age is also a prime number below 20.

How old is each member of the Jones family?

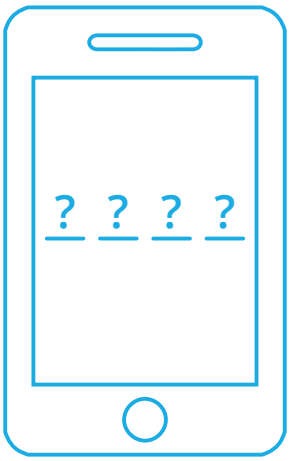
Multiplying This Way and That

Joe says that **73 × 13** will give you the same answer as **13 × 73**.

Ethan thinks that you will get a different answer.

Who is correct?
Can you explain why?

Passcode Combinations



The pass-code on my phone has four digits, e.g.

2487 2987 1087 2764

The first digit can be any digit apart from zero.

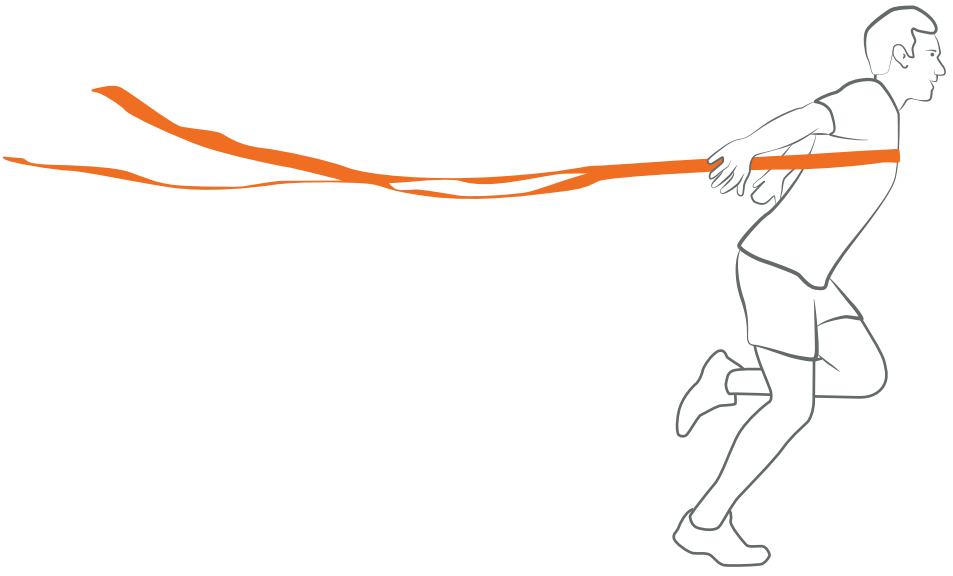
The other three digits can be any number

How many four digit numbers are there?

Triathlete Pete

A race starts with running
2 laps of a 750 metre track.
Then cycling 5 km on roads,
and next swimming 1250 metres,
and then returns to the track
for the finish.

The race is 10 km long in total.



How many laps of the track are needed to finish?

27 - No More, No less

List all the

whole number pairs

between

1 and 20

that add up to

27

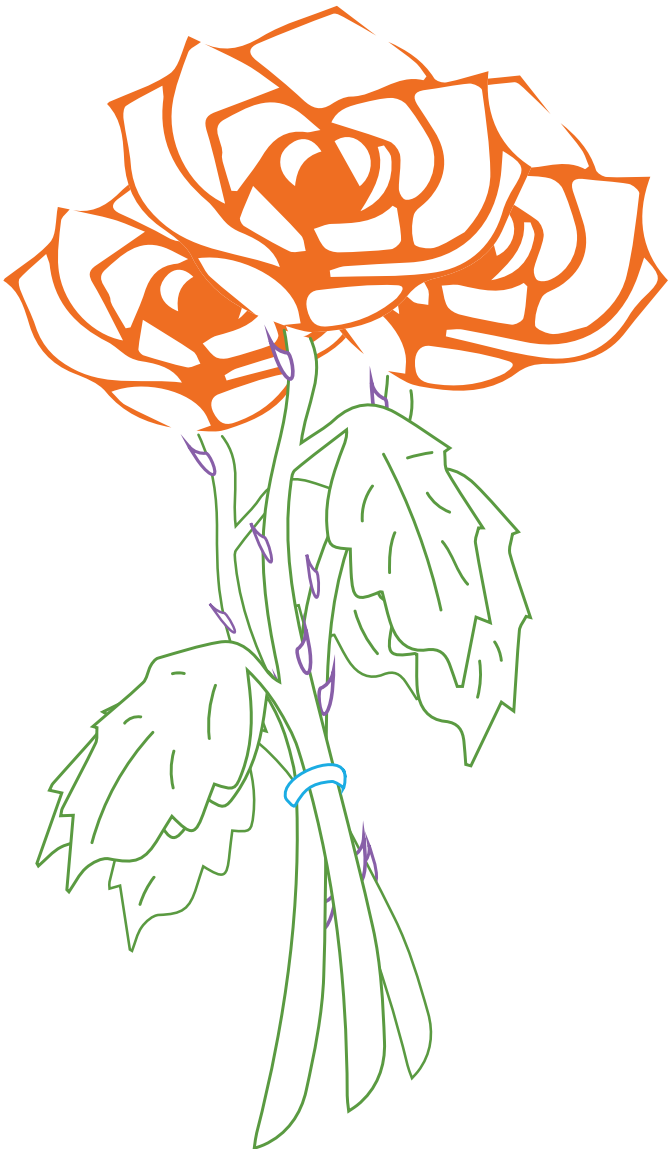
Bunch of Roses

Carnations cost £1.12 each

Roses cost £1.89 each

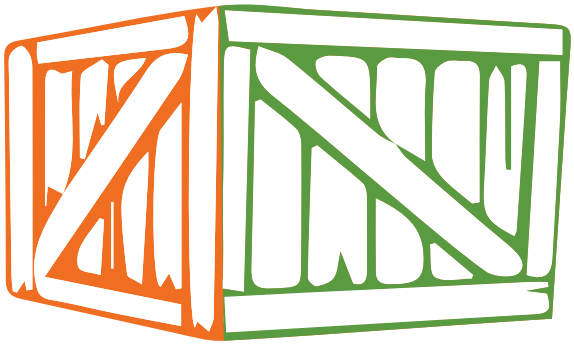
Laura buys a bunch of 12 carnations and a bunch of roses. She paid £30 and recieved £1.44 change.

How many roses did she buy?



Carlos' Coloured Crate

Carlos is painting the faces of a cube shaped crate.



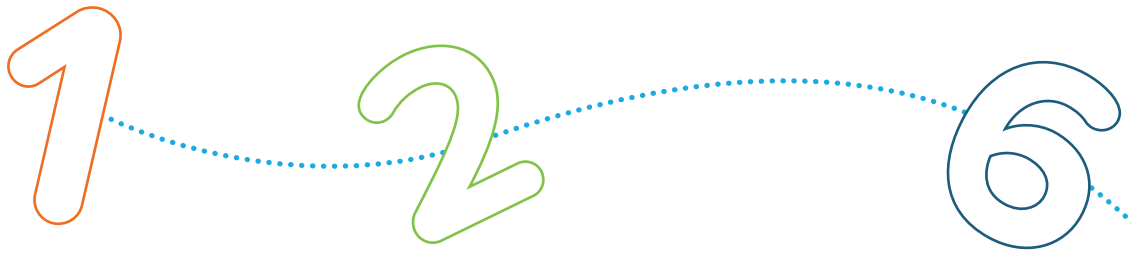
Any two faces which have an edge in common are different colours.

What is the smallest number of colours Carlos requires?

To start this problem think about the number of faces a cube is made up of and where the faces, edges and vertices meet.

Hint: It may help to draw the net of a cube.

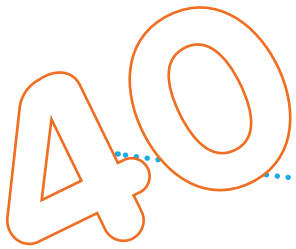
Fascinating Factors



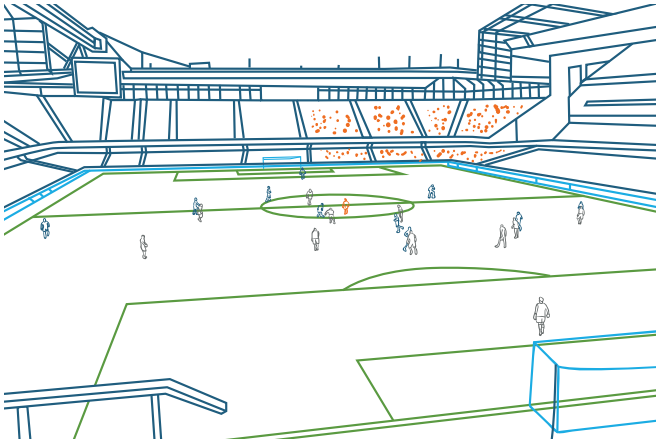
(i) Which number in the range 1-40 has the most factors?

(ii) How many factors does it have?

It is advised to use the 'factor pairs' method, but be careful.



Football or Rugby?

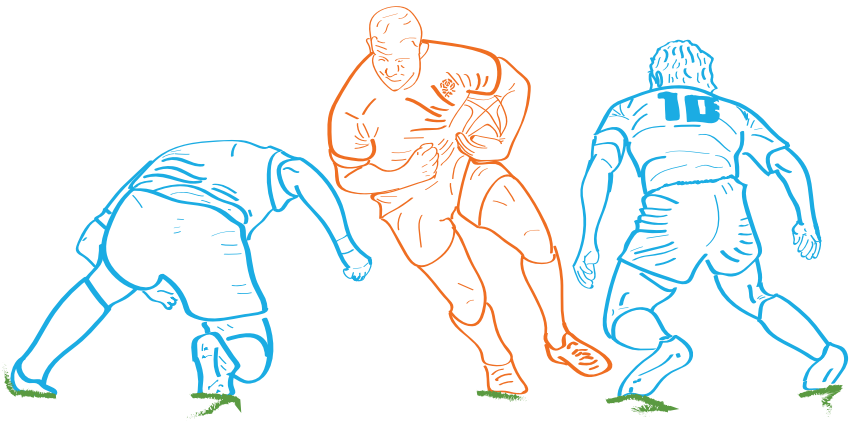


In a survey of 200 people

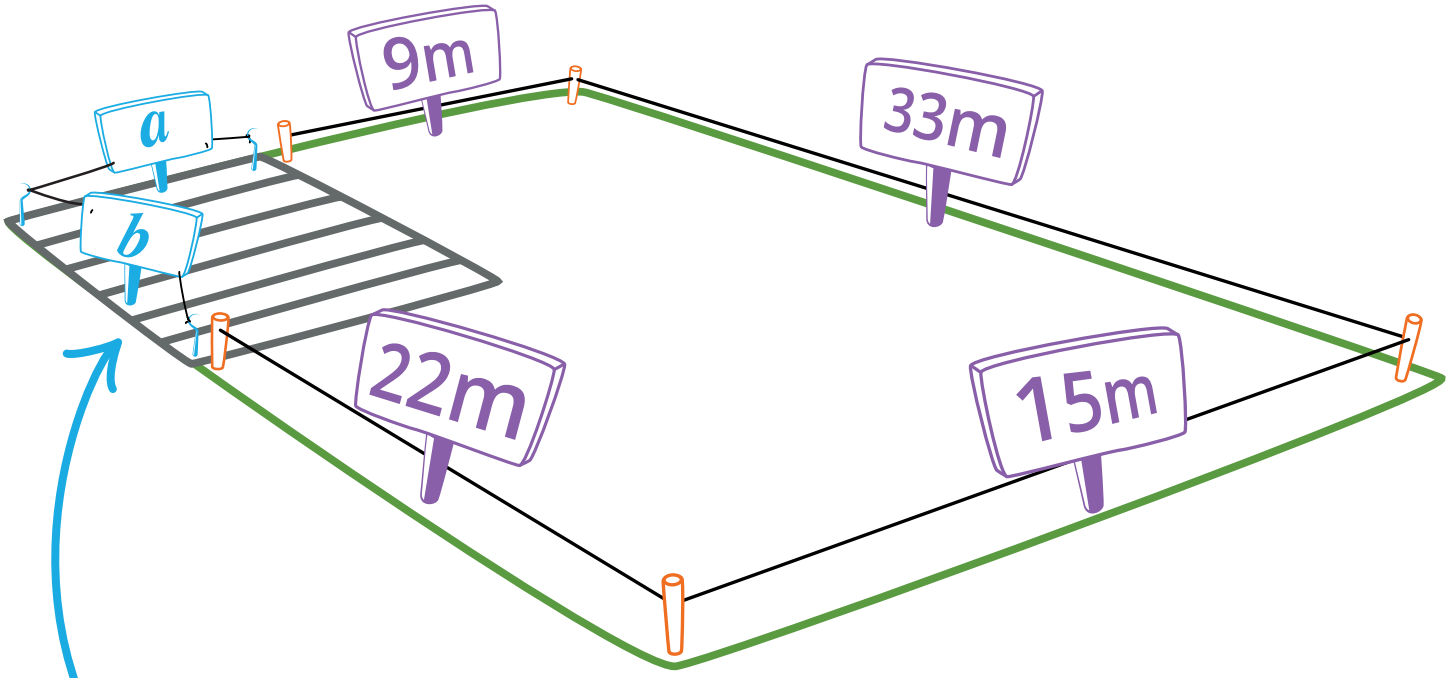
$\frac{3}{5}$ said they prefer Football to Rugby.

a) What percentage of people prefer Football to Rugby?

b) How many people in this survey prefer Football to Rugby?



Garden Greenhouse



I want to put a greenhouse in this corner of my garden.

1. What is the area of my greenhouse?
2. What is the area of the remaining garden space?

Granny's Birthday

Granny is 101 tomorrow.

So her age changes from a square number 100 to a prime number 101.

How many times has this happened before in Granny's lifetime?

Ice-cream Combinations

There are 4 flavours of ice creams
(strawberry, chocolate, coffee and vanilla)
and 3 different toppings
(sprinkles, flake and sauce).

How many different combinations are possible?

Order! Order!

Look at this sum

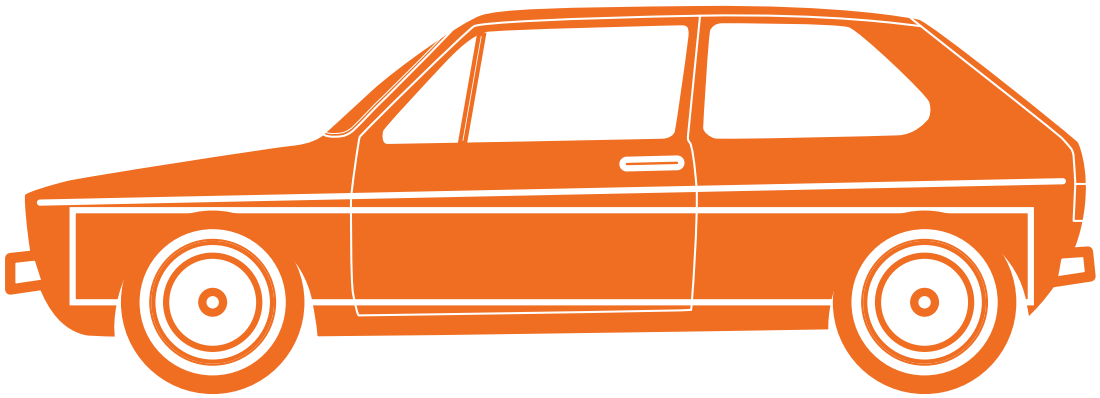
$$4 + 3^2 \times 5 - 8 \div 2$$

Firstly – what is the value of this sum?

By inserting 1 set of brackets, what is the highest number we can achieve? What is the smallest number? What are the other values that are achievable from this sum?

Could we achieve higher or lower values if two sets of brackets are allowed?

Petrol Tank



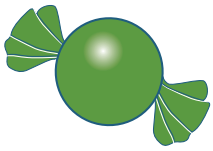
My car holds 38 litres of petrol in its tank when it is full. I used $\frac{1}{4}$ of the petrol to go to school.

a. How much did I use?

b. How much petrol was left in my car?

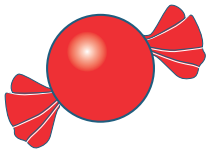
Ralph's Sweets

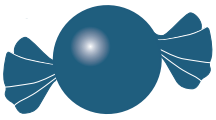
Ralph wants to put his favourite five sweets in a line.



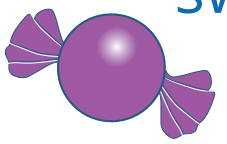
He puts the green sweet next to the red sweet but not beside the orange sweet.

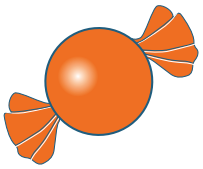
He puts the blue sweet second.





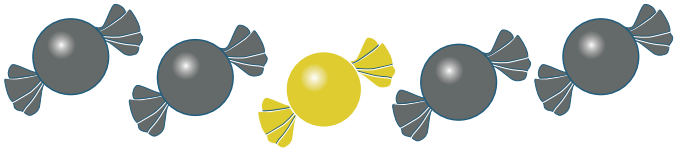
The red sweet is between the purple sweet and the orange sweet.





The blue sweet is next to the purple sweet.

Which sweet is in the middle?



Reversing Numbers

Pick three different single digit, positive whole numbers.

e.g

7, 4 & 8

Now write those numbers in order, from largest to smallest to make a new 3-digit number.

874 (A)

Take this number and call it A. Then take A and reverse is so the numbers run smallest to largest and call it B.

478 (B)

Now take B away from A and call it C

874
478 -
396 (C)

Now reverse C just like before, to get a new number D

693 (D)

Finally add C and D

396
693 +
???

What number do you get?

Two Jugs

Jug **A** holds 7 Litres of water

Jug **B** holds 5 Litres of water

How can we measure exactly
4 Litres of water?

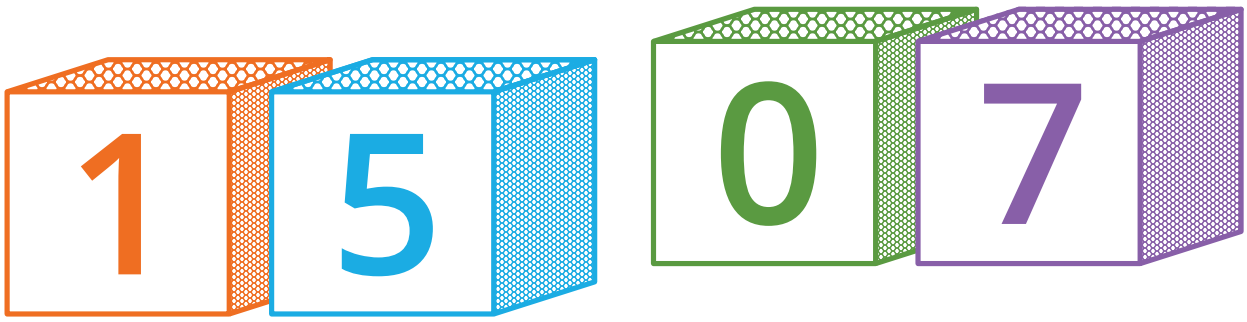
Could you do it in fewer moves?

Now try to get 8 Litres of water from a jug which holds 11 Litres and one which holds 6 Litres.

A Date with Cubes

Two wooden cubes have digits painted on each face in such a way that it is possible to display any date of the month.

For example,

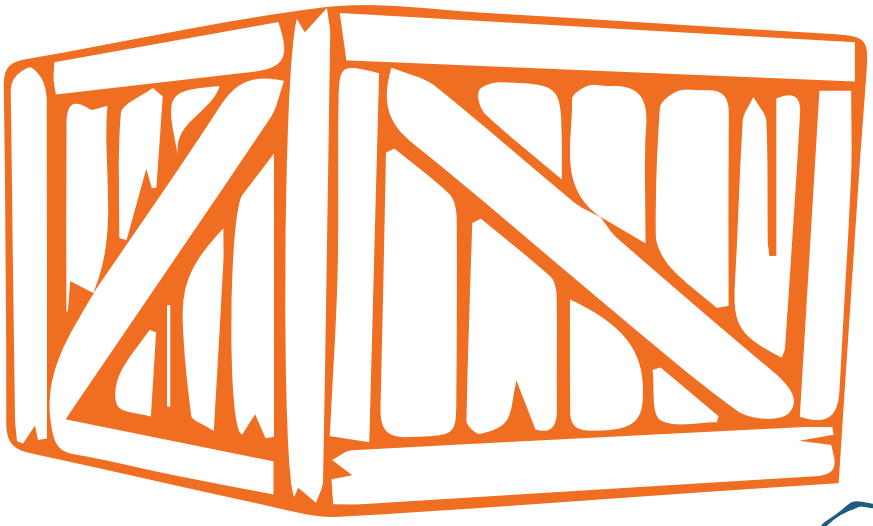


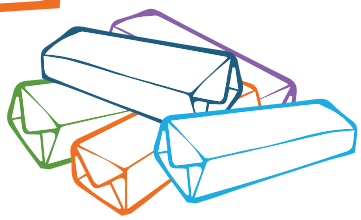
Show clearly how you would label each face to achieve this.

Chocolate Crate

A cuboid shaped crate measures 2m by 2m by 3m.

A cuboid shape chocolate bar measures 10 cm by 2cm by 1cm.



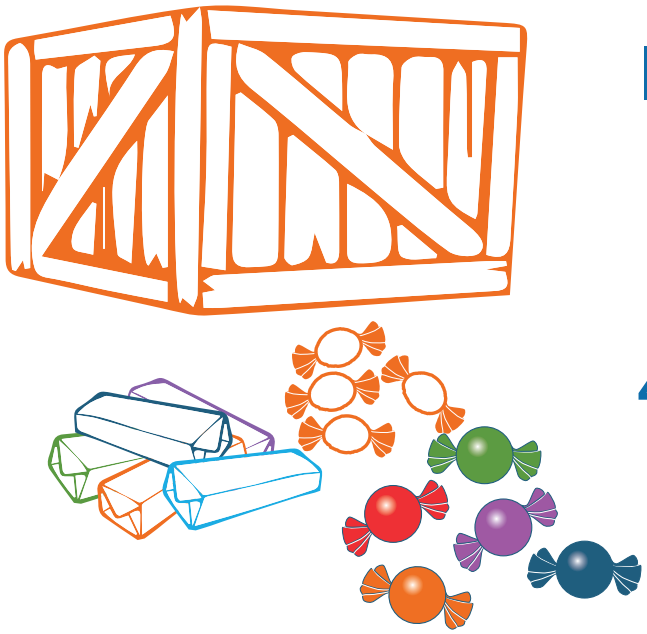


How many chocolate bars fit into the crate?

If you shared them between 20 people, how many is this each?

If you eat 1 a day, how many years would it take to eat your share?

Chocolate, Sweet or Toffee?



In a box of sweets there are 12 boiled sweets, 40 chocolates and 48 toffees.

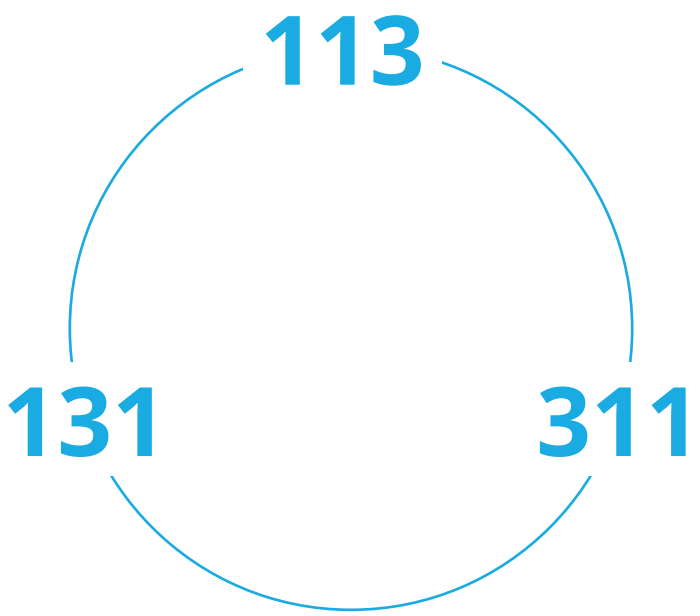
Martin's favourites are boiled sweets and is unhappy because he says that means he will never pick one.

Is Martin right or wrong?
Can you say why?

Circular Primes

A circular prime is one in which **all combinations of the digits** are also prime.

For example, 131 is a circular prime because 131, 113 and 311 are all prime.



How many **3 digit circular primes** can you find?

Consecutive 5

$1+2+3+4+5=?$

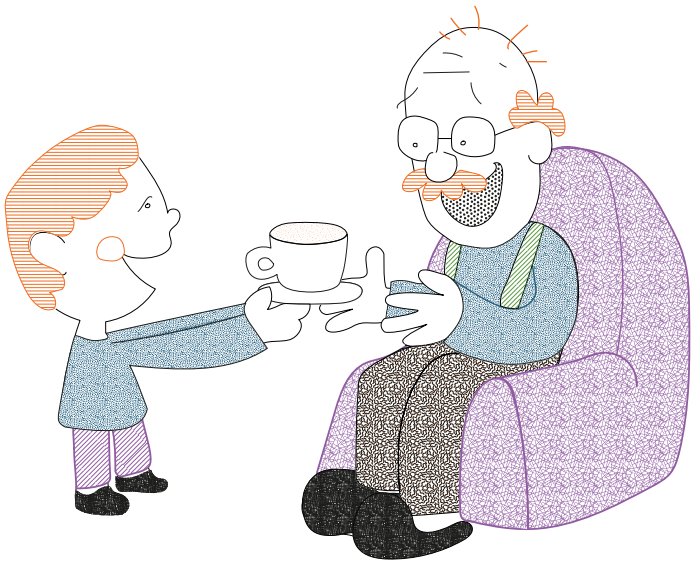
Prove that the sum of

5 consecutive whole numbers

is divisible by 5

$46+47+48+49+50=?$

Grandpa & Tom



Grandpa says he is getting younger.

He has calculated that he is now
4 times as old as his grandson Tom, but
remembers that 5 years ago he was
5 times as old as Tom was at the time.

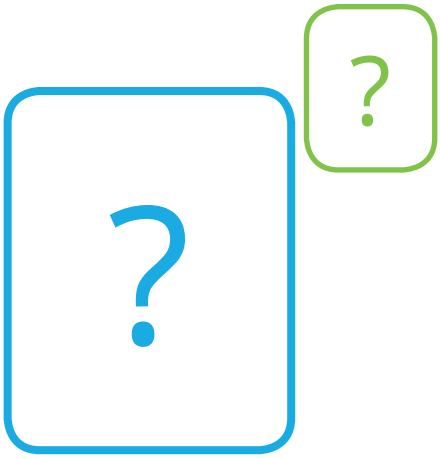
What is the sum of their ages now?

Index Indecision

Which is larger...

2^5 5^2 $?^?$

If you chose two other numbers how should you arrange them to get the biggest answer?



Is it better to put the larger number in the blue box or green box?

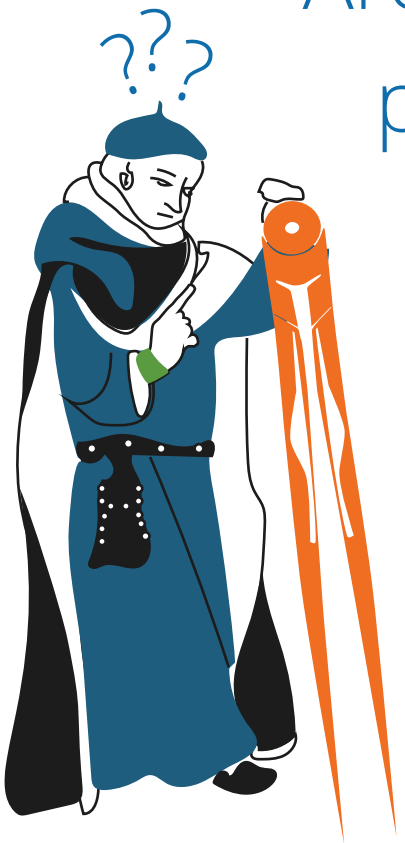
Mersenne Primes

Mersenne Primes are such that...

$$m_p = 2^p - 1$$

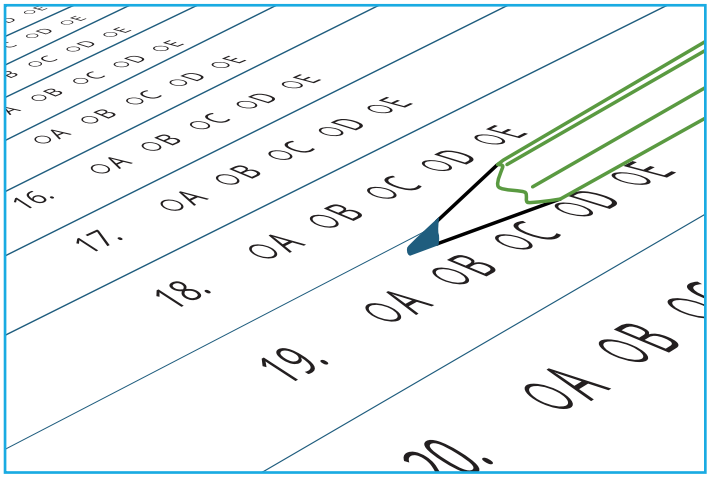
where p is a prime

Are all the values m_p prime numbers?



Multiple Choice Examination

In a multiple choice examination, Frank scores four marks for every correct answer and loses one mark for every wrong answer.



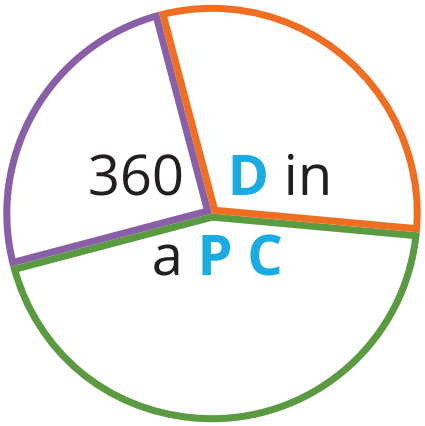
He can choose from **5** options each time.

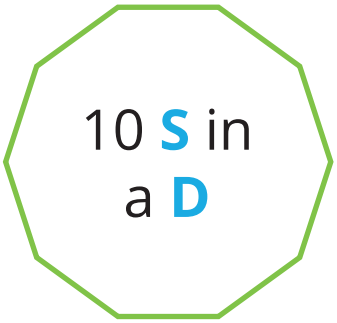
Frank attempts all 100 questions and scores 190 marks overall.

Find the number of questions he answered correctly.

One Minute Challenge 2

If a T has three S stands for "a Triangle has three Sides" What do the following mean?

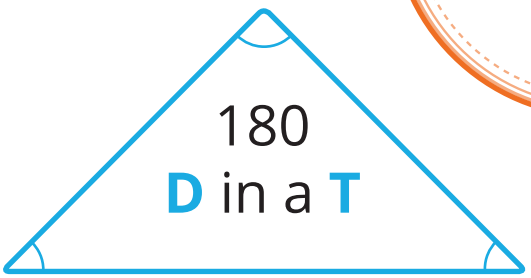




A S is
a R P

$c^2 = a^2 + b^2$
is P T

πd is the
C of a C



8 is the
S R of SF

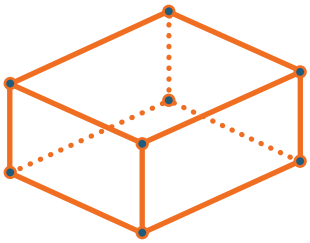
Now make up two of your own.

One Minute Challenge 3

If 360 D in a Q stands for "360 Degrees in a Quadrilateral" What do the following mean?

83 is a P N

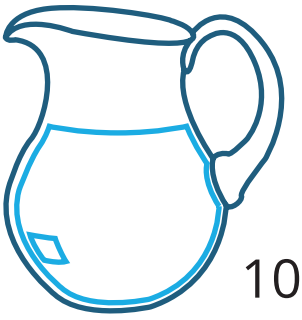
$\frac{2}{3}$, $\frac{14}{21}$ and $\frac{38}{57}$ are E F



a C has 8 V

215° is a R A

6 is the C R of 216



1000 m in a L

$\frac{1}{2} \times b \times h$ is the a of a t

540 d in a p

Now make up two of your own.

One Minute Challenge 4

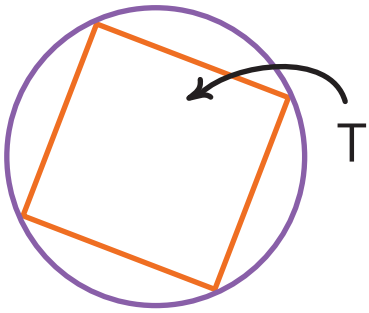
If 180 D in a T stands for "180 Degrees in a Triangle" What do the following mean?

$\frac{1}{2}(a+b)h$

...is the A of a T

1, 3, 6 and 10 are the first four TN

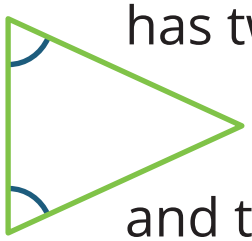
PLs never meet



This is a CQ

36 is a SN and an EN

An IT has two EA



and two ES

1000 Kg = 1T

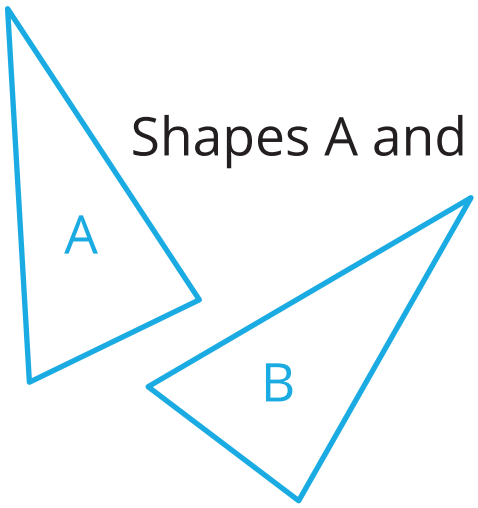
1, 2, 3, 4, 6, 8, 12, 16, 24, 48 are Fs of ? and 2 is also a PF

Now make up two of your own.

One Minute Challenge 5

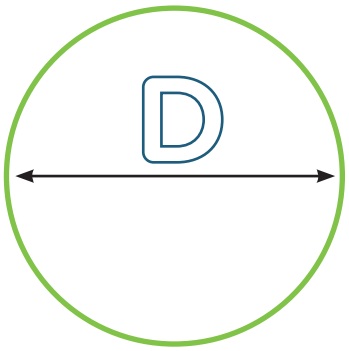
If a H has 6 S stands for "a Hexagon has 6 Sides" What do the following mean?

124° is an O a

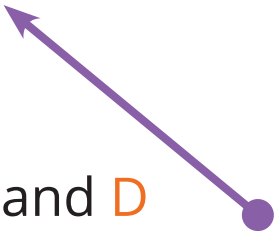


Shapes A and B are C

180° is the A of a SL



D is the distance across the centre of a C



A V has M and D

168 H in a W

$19 \times 7 \times 12 \times 0 \times 5 \times 23 = Z$

Now make up two of your own.

Square Boxes

These boxes all contain square numbers.

The largest number is next to the smallest.

Only one of the numbers has more than 2 digits.

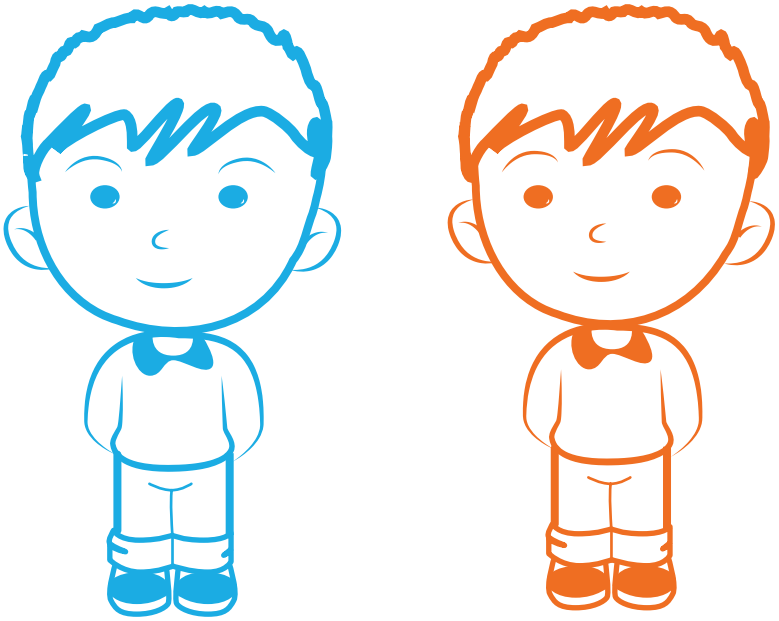
5^2 is between the only two numbers that end with a 6.

3^2 and 6^2 are next to each other.

The first two square numbers are in the last two boxes in the line.

The square root of the largest number is 14.

Ted & Tod



Twins Ted and Tod start with an identical rectangular sheet of paper.

Each of them cut their sheet into two. Ted obtained two rectangles, each with a perimeter of 40cm, while Tod obtained two rectangles, each with a perimeter of 50cm.

What was the perimeter of the original sheet of paper?

Comparing Quantities with Ratio

How is

$$\frac{4}{5}$$

different from

4:5?

How Much For Smile?

What is the value of one

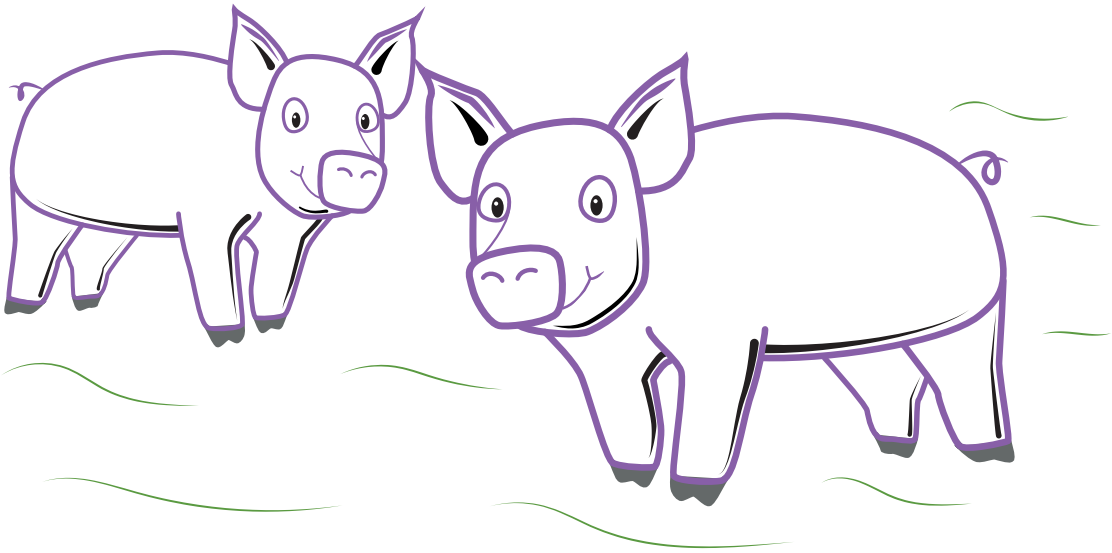
if each row and column has the given total?

			Total
			12
			11
			13
Total	12	11	13	

Can you think of a way to solve this using algebra?

Hint: Try to form equations to help you.

Penny's Pig Pen



Penny is a pig farmer.

She has plenty of land, but can only afford to buy 40 metres of flexible fencing.

Penny doesn't want to waste any fencing, but she wants to make as large an enclosed pen as possible.

What is the largest area pen that Penny can make with her 40 metres of fencing - and what shape would it be?

Extra: Do you think this is the most sensible shape for a pen?

Special Whopper Sundae


Six friends meet for ice cream at the local Ice Cream Parlour.


The **first** eats there every **day**,

The **second** eats there every **other day**

The **third** eats there every **third day**

The **fourth** eats there every **fourth day**

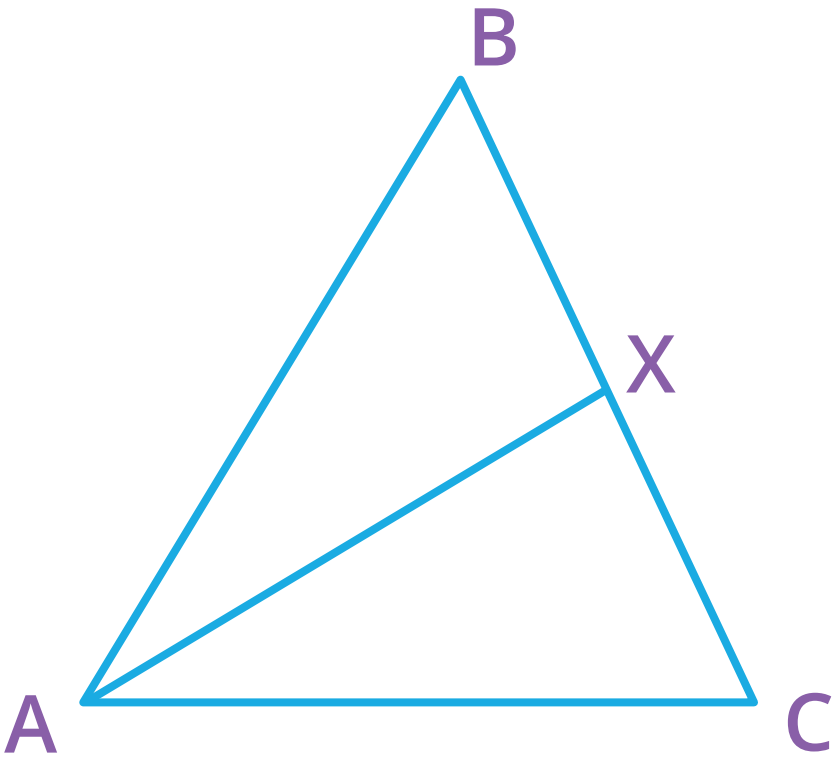
The **fifth** eats there every **fifth day**

The **sixth** eats there every **sixth day**


They agree to each have a **Special Whopper Sundae** when they are next there together.


How many days will it be

Isosceles Triangle

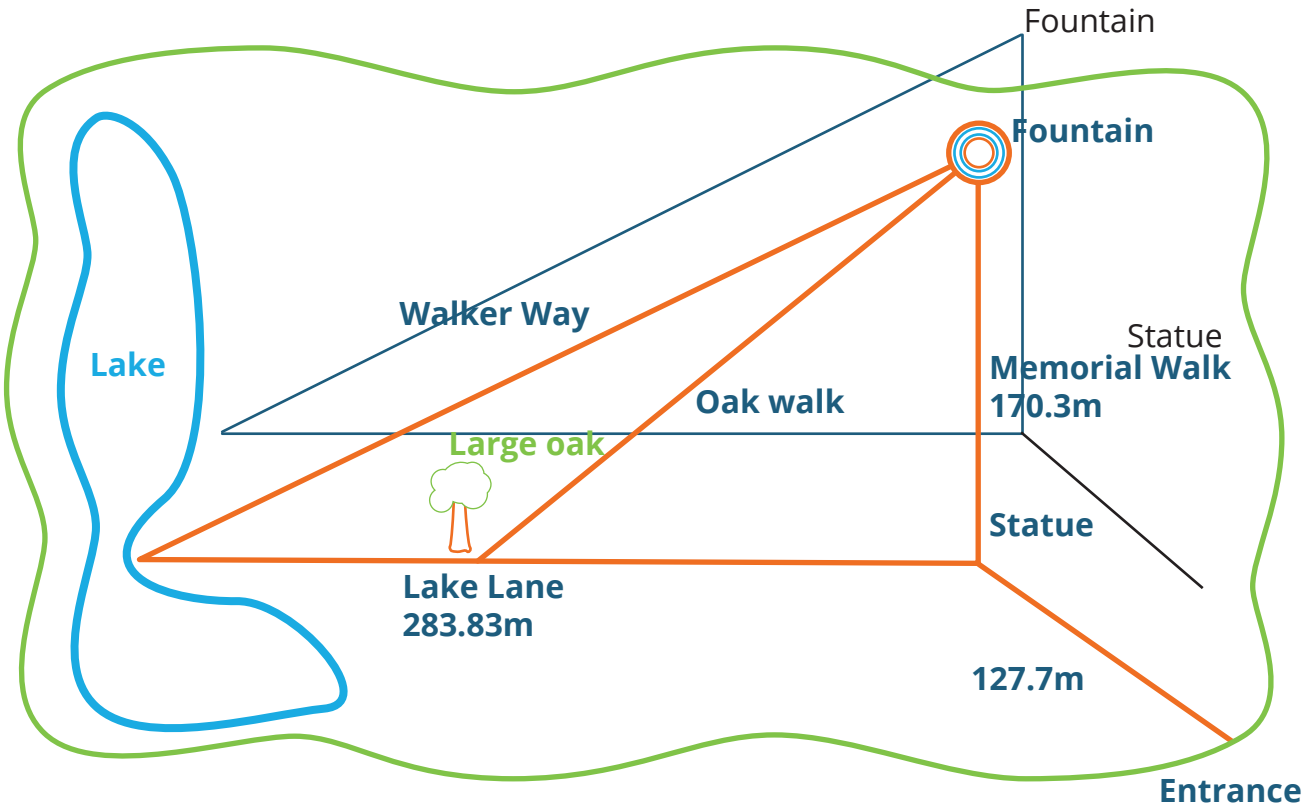


ABC is a triangle.

AB = BC
AX = BX = AC

Find all the angles in the shape.

Oak Tree Park

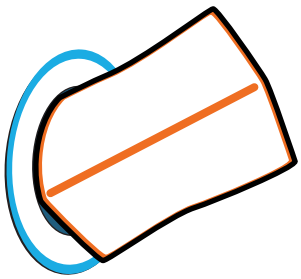


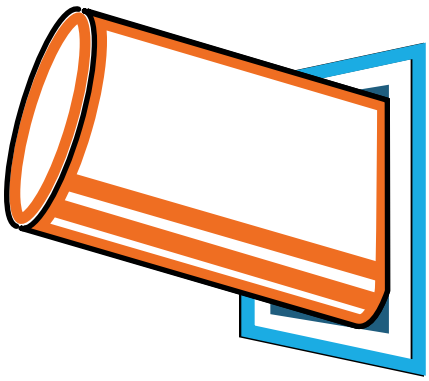
The map shows Oak Tree Park. Kavita rides her bicycle 2.1 km to the park. She wants to visit the statue, the large oak and the fountain before leaving the park. The large oak is $\frac{11}{20}$ of the way along lake lane from the statue.

a) How far does she ride around the park?

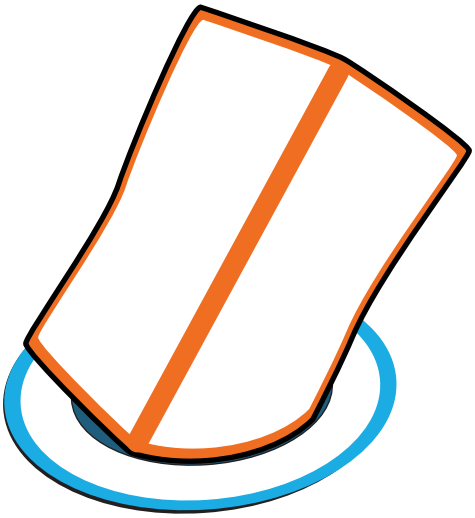
b) She takes a shortcut home and cycles for 1.7km
How far has she cycled in total?

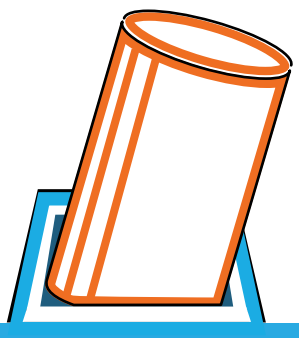
Peg of Best Fit





Which is a better fit, a round peg in a square hole, or square peg in a round hole?





Vinegar Volume

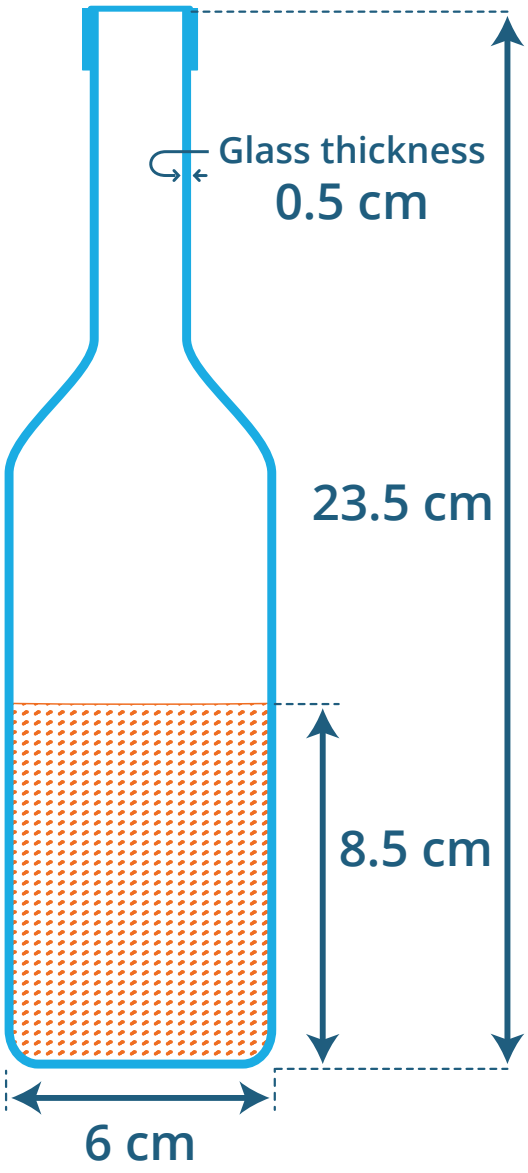
A bottle of vinegar is 23.5cm tall.

The diameter of the base of the bottle is 6cm.

The glass the bottle is made from is 0.5cm thick.

The amount of vinegar left in the bottle rises up to 8.5cm standing, or 9.5cm when upturned.

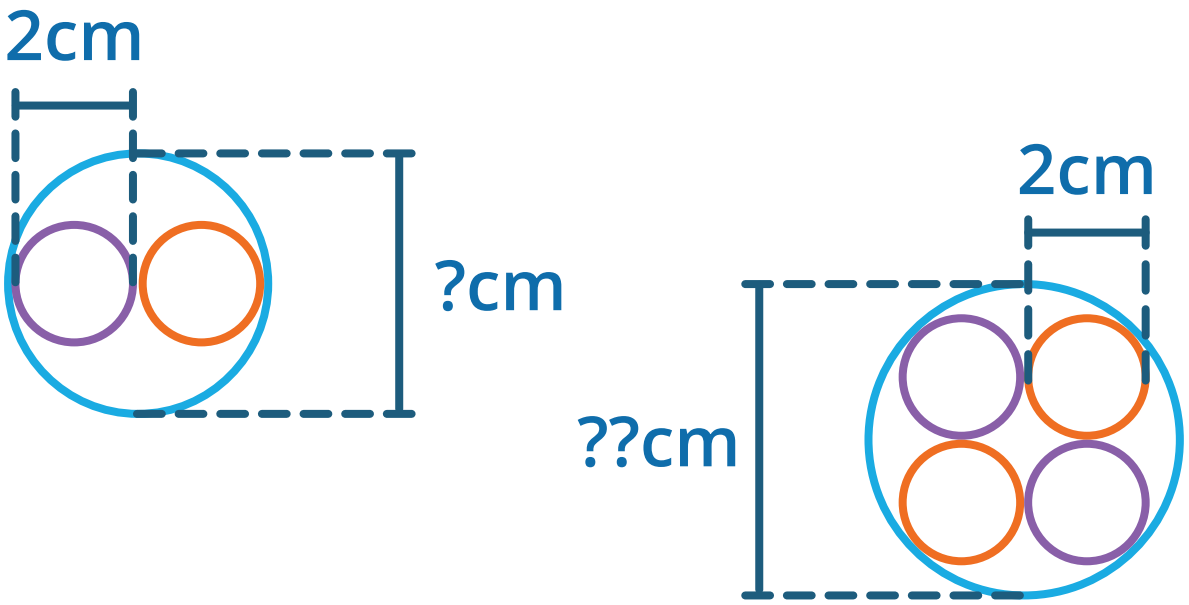
Find the internal volume of the bottle of vinegar to the nearest millilitre.



‘Broad’-band

A telephone company places round cables in round ducts.

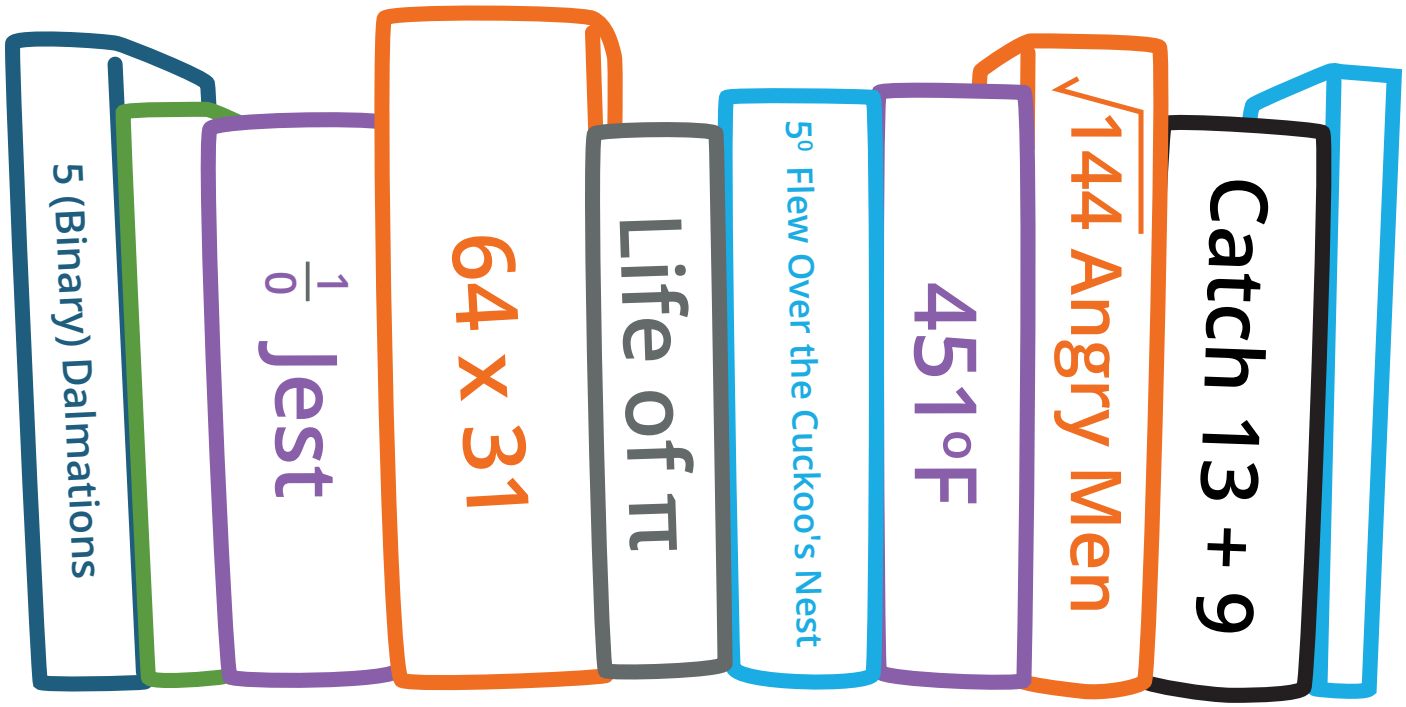
The diameter of a cable is 2 cm



What would the diameter of the duct be for two cables and four cables?

How Many Ways?

Ten books are to be arranged on a shelf.



How many different ways can they be arranged?

One Minute Challenge 6

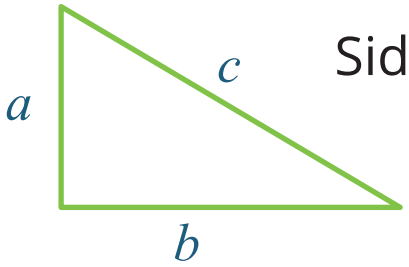
If a Q has 4 S stands for "a Quadrilateral has 4 Sides" What do the following mean?

A line just touching the edge of a C is a T

$\sqrt{7}$ is the SR of FN

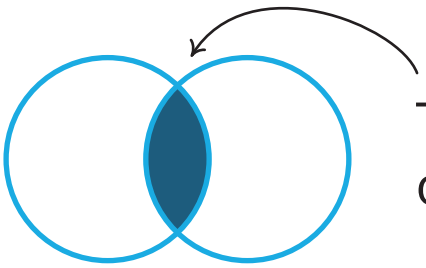
169 is a SN and an ON

$y = mx + c$ is the E of a SL



Side c is the H

A T is a PS

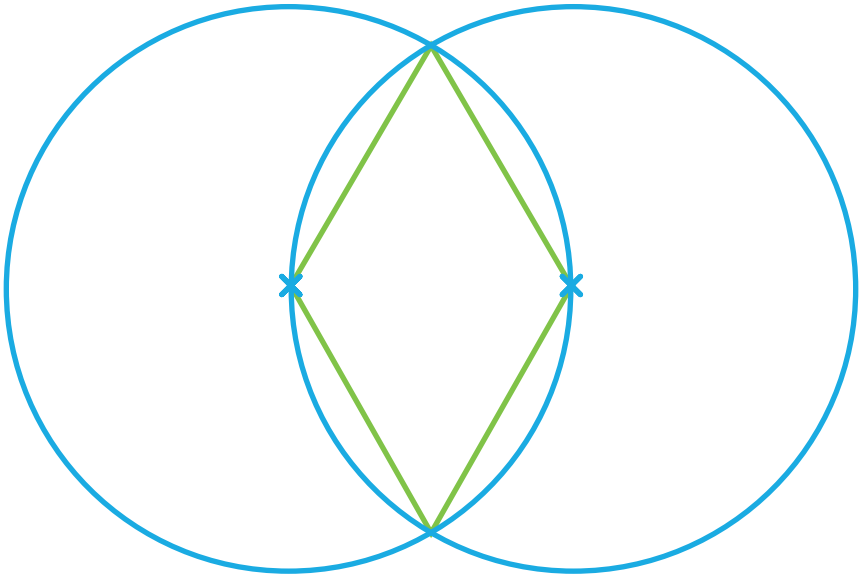


This is the I of a VD

77 is <u>not</u> a PN

Now make up two of your own.

Rhombus Overlap

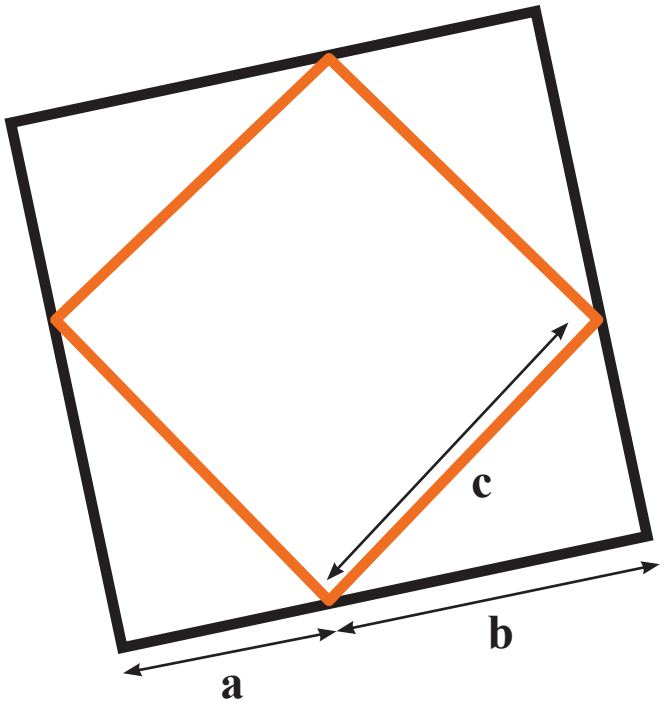


This diagram shows two circles, each of radius 2 cm.

The circumference of each circle passes through the centre of the other one.
The vertices of the shaded rhombus are the centres of the circles and their points of intersection.

What is the area of the green rhombus?

Square Within a Square



Find an expression for the area of the entire shape made by connecting a square and 4 congruent triangles.

Is there more than one way to express this?

Think of Two Numbers

Amy is thinking of two numbers.

She says:

"The highest common factor of my two numbers is 12

...and their lowest common multiple is 4620

Their sum is 552".

What are Amy's numbers?